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CCLXXXVII.

(Vol. XIII.—September, 1884.)

DISCUSSION ON

THE STRENGTH OF WROUGHT IRON STRUTS, AND ON THE STRENGTH AND ELASTICITY OF STRUCTURAL STEEL AND ITS EFFICIENCY IN THE FORM OF BEAMS AND STRUTS.*

DISCUSSION BY ONWARD BATES, ALFRED P. BOLLER, E. A. FUERTES, GEORGE H. PEGRAM, G. BOUSCAREN, THEODORE COOPER, L. L. BUCK, WILLIAM H. BURR, T. C. CLARKE, HENRY R. TOWNE, E. B. DORSEY, EDWARD P. NORTH, C. C. SCHNEIDER, PERCIVAL ROBERTS, JR., WILLIAM J. MCALPINE, O. E. MICHAELIS, AND JAMES CHRISTIE.

ONWARD BATES, M. Am. Soc. C. E.—This paper† records a series of experiments of great value to the profession, and particularly to those members engaged upon the design of wrought iron structures. Without such experiments I could not have realized the importance of placing the centre line of pressure coincident with the centre of strut. If this could be done with perfect accuracy, a round-ended strut would be as good as

* Papers by JAMES CHRISTIE, M. Am. Soc. C. E., No. CCLXXIX, page 85 (April, 1884), and No. CCLXXXVI, page 253 (August, 1884), Vol. XIII.

† Referring to Paper CCLXXIX, The Strength of Wrought Iron Struts.

a flat-ended one. We cannot rely upon obtaining this condition in practice, but we may profit by studying the effect produced by a departure from it. The results of these tests strengthen my belief of the unsafeness of many of our iron bridges, and that they do not fall down for the reason they are never loaded to their nominal capacity. The methods of securing the ends of such struts are in practice so various that I am unable to make from these experiments a table of safe loads for them.

The experiments are none the less valuable as giving us the fundamental knowledge required, and I trust will lead to further tests of full-sized specimens, such as are commonly used.

The variations of strength shown in the same strut, with different end bearings, are so great as to shake confidence in the unit strains usually allowed, and in working from the tables I would take the least results in preference to the average ones.

The tests are made upon selected specimens, straightened, and with more care bestowed on end bearings than it is practicable to obtain in actual work. In the latter, flat ends rarely have a perfect bearing, pins have play in the holes, and fixed ends are usually made by rivets through a flange or web without end bearings; the metal is placed as well with reference to convenience of fitting as to the radius of gyration, and there are frequently bends and initial strains caused by imperfect fits of the members of a structure.

With our present knowledge of the strength of struts under different conditions, the only safe practice is that of low unit strains corresponding to the lowest results of recorded tests. This becomes so much a matter of opinion that great differences in strut sections frequently occur, to the detriment of the best practice. Struts of small section have not received the same experimental attention as heavy posts and columns, and yet are fully as important. My discussion has become a plea for a continuation of Mr. Christie's experiments, the value of which I cannot overestimate.

ALFRED P. BOLLER, M. Am. Soc. C. E.—We have to thank Mr. Christie for doing some very thorough work, as far as it goes, and of illustrating still further how little knowledge we have of the compressive resistance of iron. Gordon's formula will, of course, go through some more variations, in the light of these later results. What these variations can be made to be, I will leave for other to discuss, simply express-

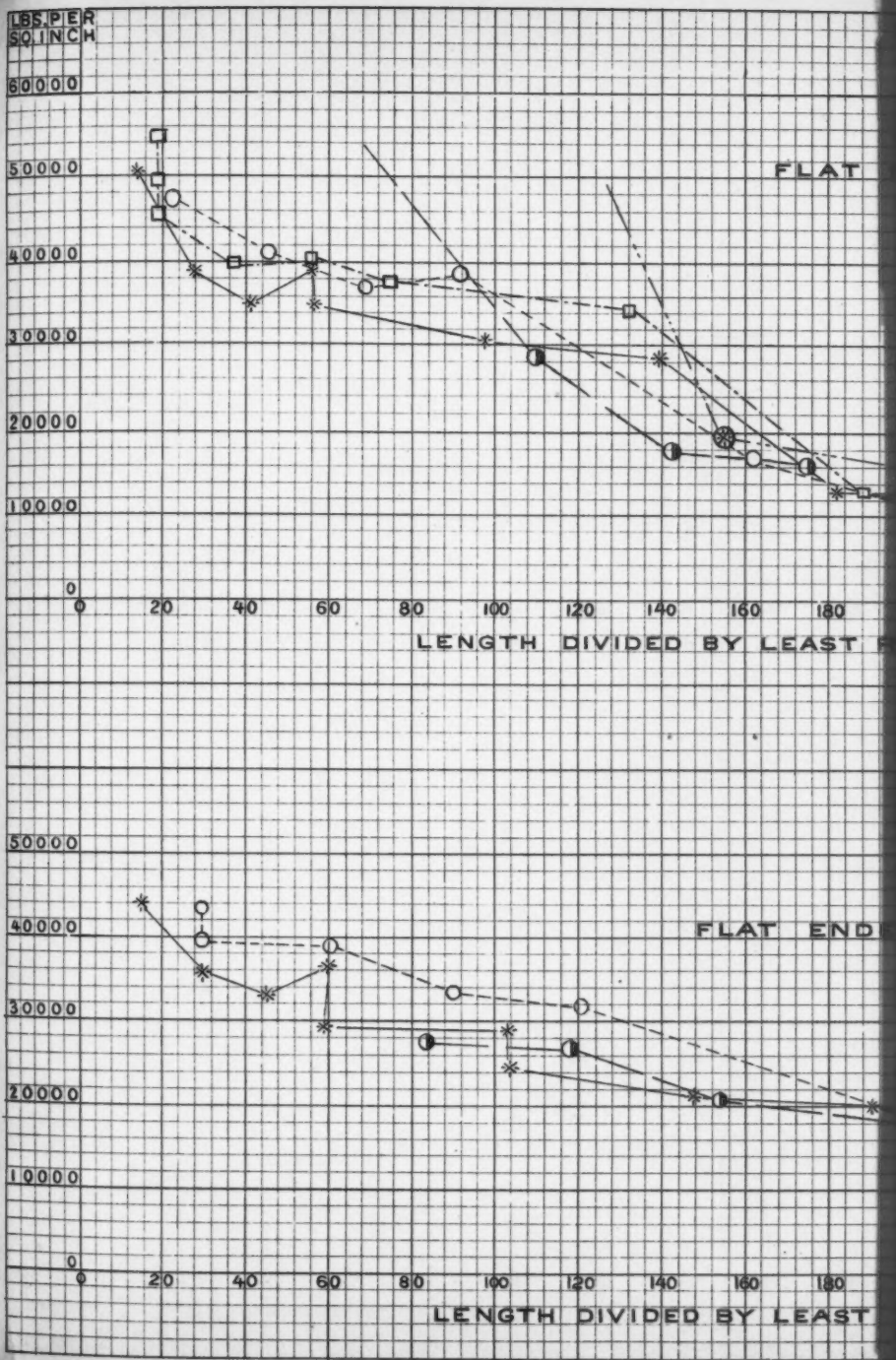
ing my conviction, that the more experiments we have in this direction, the greater will be the difficulty of arriving at any satisfactory relations of Gordon's or any other formula for the strength of struts. Any formula expressing a curve must be obedient to some law, and to discover any law in the charted results of Mr. Christie's experiments would puzzle the acutest mathematician of this or any other age. It is true, Mr. Christie has cut in a curve avoiding Scylla on one side and Charybdis on the other, and in a general way indicates the rising or falling values of $\frac{l}{r}$. What law, if any, this curve follows, I am unable to say, but it is very apparent that, at best, it will yield only approximating values. The experimental results platted in relation to this curve, are spotted very much like the stars above, and with about as much, or as little, regularity, and this, too, from specimens apparently cut from the same bar. To bring out more clearly the anomalies shown by these experiments, I have platted the flat end "tees" and "angles" on separate charts, hereto annexed, Plate XXXVIII.

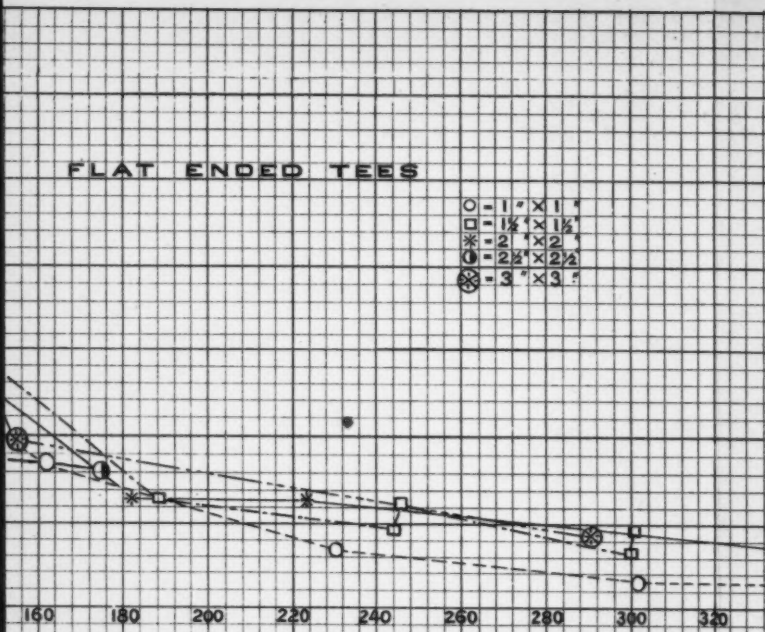
Mr. Christie's experiments have come nearer a series than anything I have seen, but I regret that while he was about it he did not carry out a series of experiments in each special size and weight of section, say at least a dozen, with lengths varying with a uniform progressive movement, say 6 inches for the lighter, and 12 inches for the heavier sections. This would give as absolute knowledge as we are ever likely to get, and settle for all time the exact value of such special forms of sections. In the experiments on flat-ended "tees," we have:

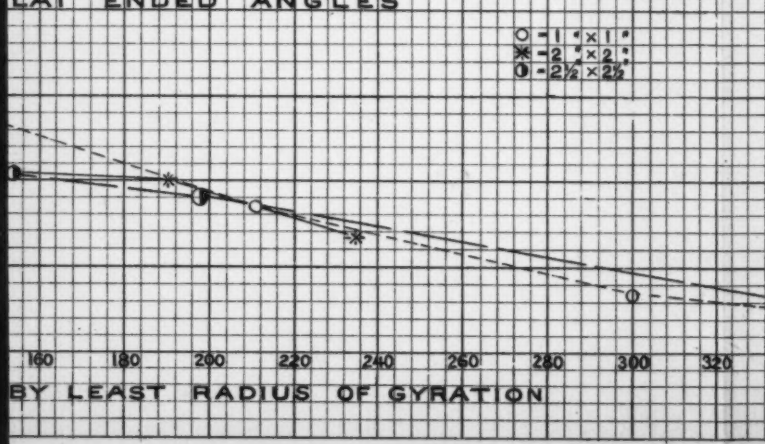
3	Exp'ts on	3 × 3	tees,	6½, 8, and 15	feet in length.
4	"	" 2½ × 2½	"	3½, 5, 6½, and 8	feet in length.
10	"	" 2 × 2	"	½, 1, 1½, 2, 3½, 5, 6½, 8 and 15	feet in length.
12	"	" 1½ × 1½	"	½, 1, 1½, 2, 3½, 5, 6½ " 8	" " "
9	"	" 1 × 1	"	½, 1, 1½, 2, 3½, 5, 6½ " 8	" " "

In the chart of these experiments some very curious results are to be noticed. For all the values of $\frac{l}{r}$ between 100 and 175, the small section 1½ × 1½ shows the strongest strut, excepting the 3 × 3, and even exceeds that section between 145 and 175. The irregularity of the connecting lines of resistance is very great, defying the application of any formula to them. Moreover, sections of the same size and length vary in

ultimate strength in a most discouraging way, as shown in the case of the three experiments on 1½ tees 6 inches long, which failed at 49 300, 45 800 and 55 000 pounds respectively. The chart of the flat-ended angles exhibits the same anomalies even in a higher degree, when the 1×1 angles exhibit a greater resistance than the 2×2 and $2\frac{1}{2} \times 2\frac{1}{2}$, between 30 and 185. The inequality of the 2×2 is extremely marked. There are not enough experiments on the other sized angles to insert in the diagram. Further, as in the case of the tees, angles of same size and ratio show great differences in ultimate strength. It is fair to assume that the other experiments, when platted, would exhibit similar incongruities, and it is difficult to account for them. The experiments were evidently made with the greatest care, and ought to be repeated on some other machine. The recorded results yielding greater strength for light sections than for heavy, is an important truth, if true, and will remodel the basis of engineering designs in a most remarkable way, and completely overturn all preconceived ideas on the strength of structures. If strength increases with reduction of sections, engineers will eventually be able to rival the famous horse that was brought down to an oat a day; but a limit had to be established somewhere—so the horse put it at that single oat and died. In an appendix Mr. Christie gives the tensile and compressive results in short specimens, round and square iron, one experiment each. The former yields results in fair harmony with previous experiments, but the latter failed (by elastic limit) at 32 000 pounds per square inch for square iron and 22 000 pounds for round iron. In each case all precautions had evidently been taken to prevent lateral flexure, and allow free play to axial bulging. The working tables, Nos. 9 and 10, of averages given by Mr. Christie, are averages from discordant results, and a mixture of all sections, and while instructive as to the *relative* effect of flat end, fixed, hinged and round ends, the absolute values must be taken with a large margin of allowance. The whole series of experiments, which represent so much care, labor and devotion, only still further point to the absolute necessity of compressive tests on a great scheme, and the Watertown authorities could do no better service than to devote their machine to such work, and establish the working value of all forms of struts and compressive members used in the art of construction. In my judgment, no formula will ever give satisfactory results, and dependence must be placed upon experimental charts, which will express extreme values for all sec-


LBS. PER SQ. INCH
60000
50000
40000
30000
20000
10000
0
0
20
40
60
80
100
120
140
160
180
FLAT
LENGTH DIVIDED BY LEAST
50000
40000
30000
20000
10000
0
0
20
40
60
80
100
120
140
160
180
FLAT END
LENGTH DIVIDED BY LEAST


FLAT ENDED TEES
○ - 1 " × 1 "
□ - 1½ " × 1½ "
✳ - 2 " × 2 "
◑ - 2½ " × 2½ "
⊗ - 3 " × 3 "
160
180
200
220
240
260
280
300
320
BY LEAST RADIUS OF GYRATION


LAT ENDED ANGLES
○ - 1 " × 1 "
✳ - 2 " × 2 "
◑ - 2½ × 2½
160
180
200
220
240
260
280
300
320
BY LEAST RADIUS OF GYRATION

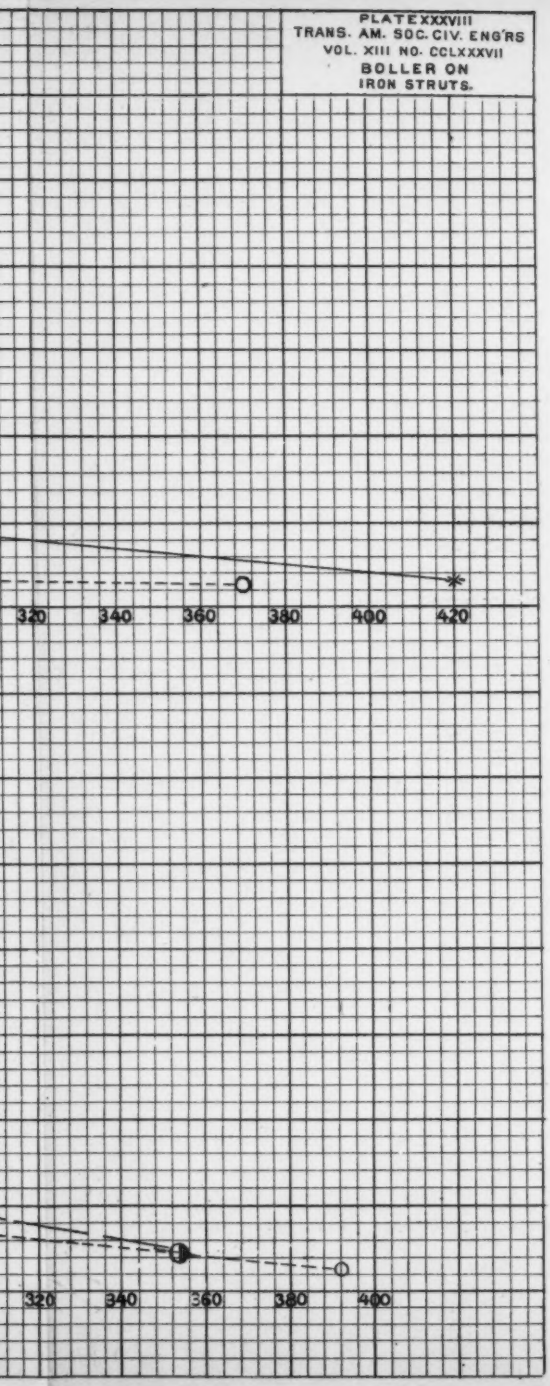


PLATE XXXVIII
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BOLLER ON
IRON STRUTS.
320
340
360
380
400
420
320
340
360
380
400

tions referred to $\frac{l}{r}$, progressively determined. Isolated experiments give very little information, further than the value of the particular piece. Table No. 7 gives the following:

4	exp'ts,	5-inch	channels,	3½, 6½, 5, and 8 feet long.
1	"	6 "	"	15 feet long.
1	"	8 "	"	8 " "
4	"	4 "	"	3½, 5, 6½, and 8 feet long.
4	"	3 "	"	3½, 5, 6½, " 8 " "
4	"	2 "	"	3½, 5, 6½, " 8 " "

1 6-inch beam, 2 5-inch beams, and 2 4-inch beams.

Such tests are insufficient to form a table, or determine the value of intermediate or greater lengths. We simply learn what those few specimens bore.

It is a great advance when we discover and manfully own up our ignorance; it is a precursor of a movement toward right information, and I know of no experiments that bring this home more forcibly than those of Mr. Christie—an indebtedness I, for one, am glad to acknowledge. There is no department of engineering that has not common ground with every other in this important branch of experimental knowledge, and united effort should be made to have such a systematic series of experiments instituted as will settle for all time the endless uncertainties that surround the whole subject.

E. A. Fuertes, M. Am. Soc. C. E.—I think that the areas of cross-section should be obtained by direct measurement, instead of deriving them from the weight and length of the bars, especially when the specific gravity of the material is not determined. There are far less mechanical difficulties in the former method, with special advantages that are entirely taken for granted in the method employed; while the true areas of cross-section are important elements in these experiments.

The reason why an accurately centred straight bar behaved as a flat-ended strut, when hinged, must be due to the friction developed by pressure on the bearing of the hinge; and, on the other hand, the early failure of flat-ended struts was due, probably, to the want of parallelism between the planes of the extremities of the bar; or to either one or

both of these planes being warped surfaces, instead of perfect and parallel planes.

The frequently observed behavior of bars which produced rotation of the hinge as soon as loaded, must have been due to curvature in the axis of the bar, imperfect centring, or the reason given above. The load on a straight, well-centred bar will at once develop friction at the hinge, which will be proportioned to the normal pressures on its surface. When the resultant of these pressures, owing to flexure in the bar, becomes so oblique that it cannot counteract the moment of the friction, rotation at the hinge must begin; and soon after the bar must spring suddenly out of the machine if its elasticity has not been crippled; the force with which the bar will spring depending on the elasticity remaining in the bar and the conditions affecting the friction of motion at the hinge. Lubricating the bearings of the hinge will necessarily lower the limit of loading at which the springing out will take place.

In the opinion of Professor Church, it should not be confidently stated that "the deflection under these conditions more resembled that of fixed-ended than of flat-ended specimens;" for, in fact, if the friction of the bearing is such as to prevent its rotation, the experiment, up to this point, may be considered as not being made on a hinge. The fact that in no instance could the springing out be produced by round-ended struts may be explained by the fact, that, under the conditions described, the friction at the hinge does not exist in the experiments on round-ended bars between fixed parallel plates, rolling friction only being developed as the points of contact of the rounded end with the machine plates shift sideways in the direction in which flexure takes place. The springing out may, however, take place if the radius of curvature at the ends is large; and the tendency to spring out will diminish as the ends approach a semi-sphere; for the bar, at this limit, may be bent 180° without disturbing the verticality and parallelism of the lines joining the points of contact of the plates with the extremities of the bars.

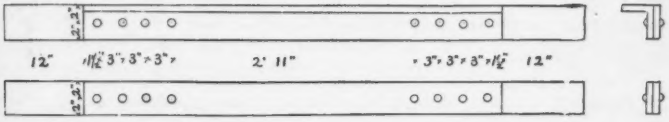
When a bar is very long in proportion to its radius of gyration, it will fail with a comparatively light load, without permanent injury to the bar. It seems to me proper that the load required to produce failure under these conditions should be given a name other than ultimate load. The latter term should be restricted to its admitted bearing on the elastic limit of the material acted upon.

GEORGE H. PEGRAM, M. Am. Soc. C. E.—I desire to emphasize what Mr. Christie has said about the effect of eccentricity of stress, or the non-coincidence of the axis of stress with that of the member.

Angles and tees used as struts in practical constructions are almost invariably connected with pins or rivets in one leg. The eccentricity is thus made much greater than that existing in the Pencoyd experiments, and the reduction of strength is correspondingly increased.

To illustrate this effect, I have, in the absence of facilities for making a compressive test, had the following tensile test made:

From a lot of angles, of which specimen tests ranged from 46 000 to 50 000 pounds per square inch, a 4 x 4 x ⅜-inch angle was taken and two 4′ 8″ lengths cut therefrom. One of the legs of one of these pieces was sheared off, leaving a 4-inch by ⅜-inch flat. To each end of both pieces a 4-inch by ⅜-inch flat was attached, with four ¾-inch rivets at 3-inch pitch in the centre line of the leg, as shown in the following sketches:



In the process of shearing, the flat was somewhat bent and had to be cold-straightened; barring this, the conditions of the pieces were as nearly the same as could be made. The grippers of the testing machine were attached to the 4 x ⅜-inch flats and the pieces pulled apart.

Both pieces broke at the first rivet, the flat parting at once, and the angle first breaking in the section between the rivet and the edge of the leg and then gradually tearing apart. The ultimate strength of the flat was 40 100 pounds, and that of the angle 23 030 pounds per square inch of net section. The flat would doubtless have stood more but for the injury done in straightening.

In a compression test, the difference in strength would be much greater, on account of the greater tendency to distortion.

I should have some hesitancy in presenting an isolated test, except to illustrate a point, but its accordance with previous experiments assures me that it will not be misleading.

A series of tests made with the same thoroughness as those which

Mr. Christie details, in which the angles and tees are made to bear on one leg, would be especially valuable.

G. Bouscaren, M. Am. Soc. C. E.—I regret very much that I cannot find time to do justice to this interesting subject. I can only say, after a hasty perusal of the paper, that my own experience in that line confirms the results of Mr. Christie's experiments as regards pin-bearing columns. I have found that the ultimate strength of such columns is always in excess of that of the same column with rounded ends; the excess increasing with the diameter of the pin and the degree of close-fitting of the same. I observe that the flat-bearing struts in Mr. Christie's experiments were generally deflected, as per curve **A B**, which obtains for round-ended struts, whilst all flat-bearing posts, which I have tried, have deflected, as per curve **C D**, which obtains for fixed-ended posts. To my mind this may be attributed to the small diameter of the struts used by Mr. Christie.

Theodore Cooper, M. Am. Soc. C. E.—The experiments made by Mr. Christie upon wought-iron struts are a most valuable contribution to our knowledge of the strength of materials.

In addition to my personal knowledge of the author's careful methods of investigation, the tests, as reported, bear strong internal evidence of having been made with extreme care.

The method of carrying out complete series, with different end connections, upon iron which is practically identical in character, has vastly increased the value of the results. The author has not remained satisfied with simply noting anomalous results, nor, as is done too frequently, rejecting them entirely as non-reliable, but has proceeded upon the true scientific method of varying the conditions to discover why the results are anomalous. He is thus able to show conclusively that slight changes in the direction of the lines of applied forces produce great changes in the results. By interchanging the different sizes of ball and socket joints upon the same bars, he proves very conclusively the influence of the size of such connections upon the compressive resistance of pivoted end struts.

By carrying his series far beyond the limits of practical ratios of lengths to transverse dimensions, he has given us a more complete knowledge of the action of struts of high ratios than we have before possessed. I think especial credit is due the author and the Pencoyd Iron Works for the complete manner in which these tests have been made, the liberal spirit in which they have been executed, without Government aid, and for giving the results freely to the engineering profession. It is to be hoped that the spirit with which they have been animated may similarly affect other manufacturers, and that the testing may fall into the hands of as careful and painstaking an investigator.

The method suggested by me in the discussion of the tests of Phœnix columns (Paper CCXXXIII, Vol. XI), of referring all ratios to the least radius of gyration, instead of the least dimension, is especially applicable to these tests, as no other method could show a fair comparison between such various forms as angles, tees, channels and tubes.

It has been adopted by the author in plotting and tabulating his tests, and gives very good comparative results. In considering the tests upon hinged end struts, as given by the author, it is very necessary to bear in mind the *relative* sizes of the ball and sockets to the transverse dimensions of the struts. The strength of the struts with ball and sockets of 1 inch or 2 inches diameter, is not to be taken as the strength of struts with these sized joints, regardless of the dimension of the struts, but rather as the strength of *similar* struts with joints of the same *relative* sizes.

Upon a diagram (Plate XXXIX), I have plotted the various tests of the author for hinged jointed struts in a manner to show the relative proportion between the ball and sockets and the least transverse dimension of the struts.

Upon the same diagram I have plotted the tests made at the Watertown Arsenal upon the pin-ended columns (large sizes) of the Detroit Bridge Company. These columns were made of two channels latticed on the sides. The channels were 6, 8 and 12 inches in width, and had pins from 3 to 3½ inches in diameter. The relative sizes of these pins to the channels of the columns are also shown upon the diagram.

It will be seen that the ratios of the balls to the transverse dimension of the struts varied in the author's tests from unity to one-half, or, in other words, that the diameters of the balls in some cases equaled the width of the strut and varied from this to one-half the width of the strut.

In the Watertown experiments, the pins varied from $\frac{7}{12}$ to $\frac{1}{2}$ of the width of the posts.

From an inspection of the diagram, the influence of the size of the pins relatively to the width of the struts is shown very conclusively. This method of plotting the tests also shows the reason why many of the tests with 1-inch balls exceeded those with balls of 2 inches in struts of the same ratios of length to least radius of gyration. The 1-inch balls were relatively larger than the 2-inch balls. In discussing the tests of Phœnix and other columns (Paper CCXXXIII, Vol. XI), I pointed out the probable fact that the "strength of pin-ended columns would be found to be somewhat governed by the diameters of the pins and the closeness of their fit to the pin-holes," and "in believing that at some point at or beyond 120 radii of gyration, the sections of the columns become so small in reference to the length, that the resisting moment at the ends diminishes rapidly and reduces to nothing finally."

"The round-ended column is the only one which would be free from this resistance to flexure"—the resistance due to the more or less fixedness of the ends in square and pin-ended columns.

"We would probably find that those with round ends would give the closest correspondence to our theoretical formulæ, and that the others could be derived from it by adding a feature representing the resisting influence of the ends."

"Such formulæ would not only be rational, but would point out more definitely the effects of the ends than those we now have in use, where as much weight is given to the squareness of the ends upon a column 100 diameters long as to a column 10 diameters long."

The present tests show very conclusively the correctness of these views. The following table (page 277) is formed by tabulating the *differences* between the strength of round-ended tees (taken from the author's table of averages, No. 9), and that of the other more rigid connections. For this purpose we have taken flat-ended tees, hinged-ended tees, and fixed-ended angles. This last is not exactly a correct comparison, but as there were no fixed-ended tees, we have had to use this series. While not strictly correct, it will serve as pointing out the relative effect near enough for the present discussion.

The values in this table represent the relative influences of the end connections, referred to round-ended columns, which are practically

TABLE SHOWING INFLUENCE OF END CONNECTIONS AT DIFFERENT RATIOS, $\frac{l}{r}$

$\frac{l}{r}$	FLAT-ENDED TEES.	HINGED-ENDED TEES.	FIXED-ENDED ANGLES.
20	5 000	3 000	1 000
40	5 500	4 500	1 500
60	7 500	5 500	3 500
80	10 000	6 000	7 000
100	11 000*	6 500*	9 500
120	10 500	6 000	11 500
140	10 200	5 700	12 700
160	10 500	6 000	13 500*
180	9 500	5 000	12 500
200	8 000	4 500	11 500
220	7 000	3 500	10 000
240	6 700	2 700	8 700
260	6 200	2 200	7 200
280	5 300	2 300	6 800
300	4 200	2 200	6 200
320	3 000	2 000	5 500
340	2 400	1 900	4 900
360	2 100	1 600	4 600
380	1 800	1 300	4 100
400	1 500	1 000	3 700
420	1 200	900	3 500

* Maximum.

free from such influence. These values are also plotted upon the diagram (Plate XL).

From these it is seen that the maximum influence of the ends allows between 120 and 160 radii of gyration.

For this reason we should expect, what is shown upon the first diagram (Plate XXXIX), that the tests at this point would cover a wider range of strength than elsewhere; as the more or less rigidity of the end connections would exert its maximum influence at this point. On the diagram (Plate XXXIX) we see that the tests upon the bridge columns of the Detroit Bridge Company run a little higher than tests upon similar ratios made by the author. This might be expected, for the following reasons: 1st, the author lubricated his balls and socket joints; 2d, the radius of the moment of friction would be relatively less upon a spherical joint than upon a cylindrical one.

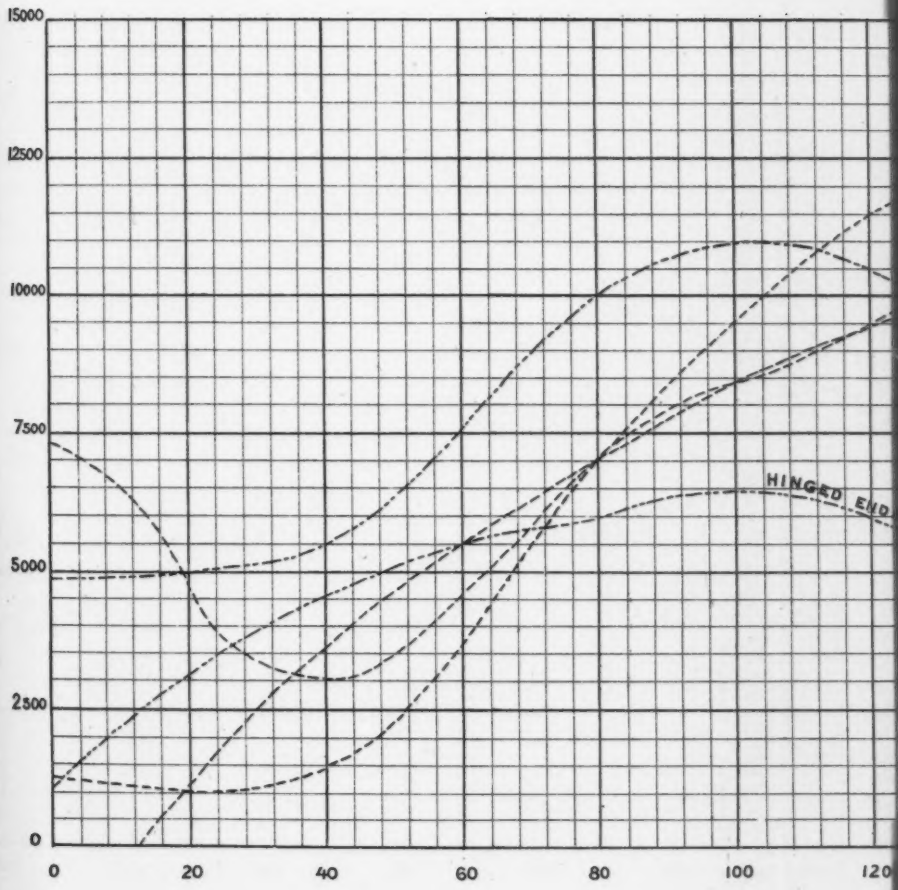
Referring to the author's diagram, No. 6 (Plate XXVIII), for round-ended tees, it will be seen that his average curve is a very regular one, and corresponds very closely to a parabola: $a\left(M-\frac{l}{r}\right)^2+c.$

[$.7\left(260-\frac{l}{r}\right)^2+3\,800$ corresponds very closely to the curve up to 260 radii of gyration.]

The curve of these tests is the most regular curve of the whole series, and, as we have stated before, this should be the nearest approach to the theoretical law of the resistance of thin columns. The others should differ from this by the effect due to the particular end resistances of each class.

It must be borne in mind that the figures in table (page 277), used in plotting the diagram Plate XL, are the author's *general averages*, which lump the tests with the various sized balls and sockets.

As many of these are far in excess of the usual relative proportions employed in practical bridge work, it is desirable that new averages, considering the relative sizes of the balls and pins, should be determined. I have been prevented by more urgent duties from carrying out this investigation, so necessary to show the relative influence of the sizes of the pins. In practice, our bridge pins will average from about $\frac{1}{3}$ to $\frac{1}{4}$ of the transverse dimensions of the posts, which is about the lower limits of the author's experiments. From the great effect of decentralizing the line of applied forces upon columns of initial, though minute, bends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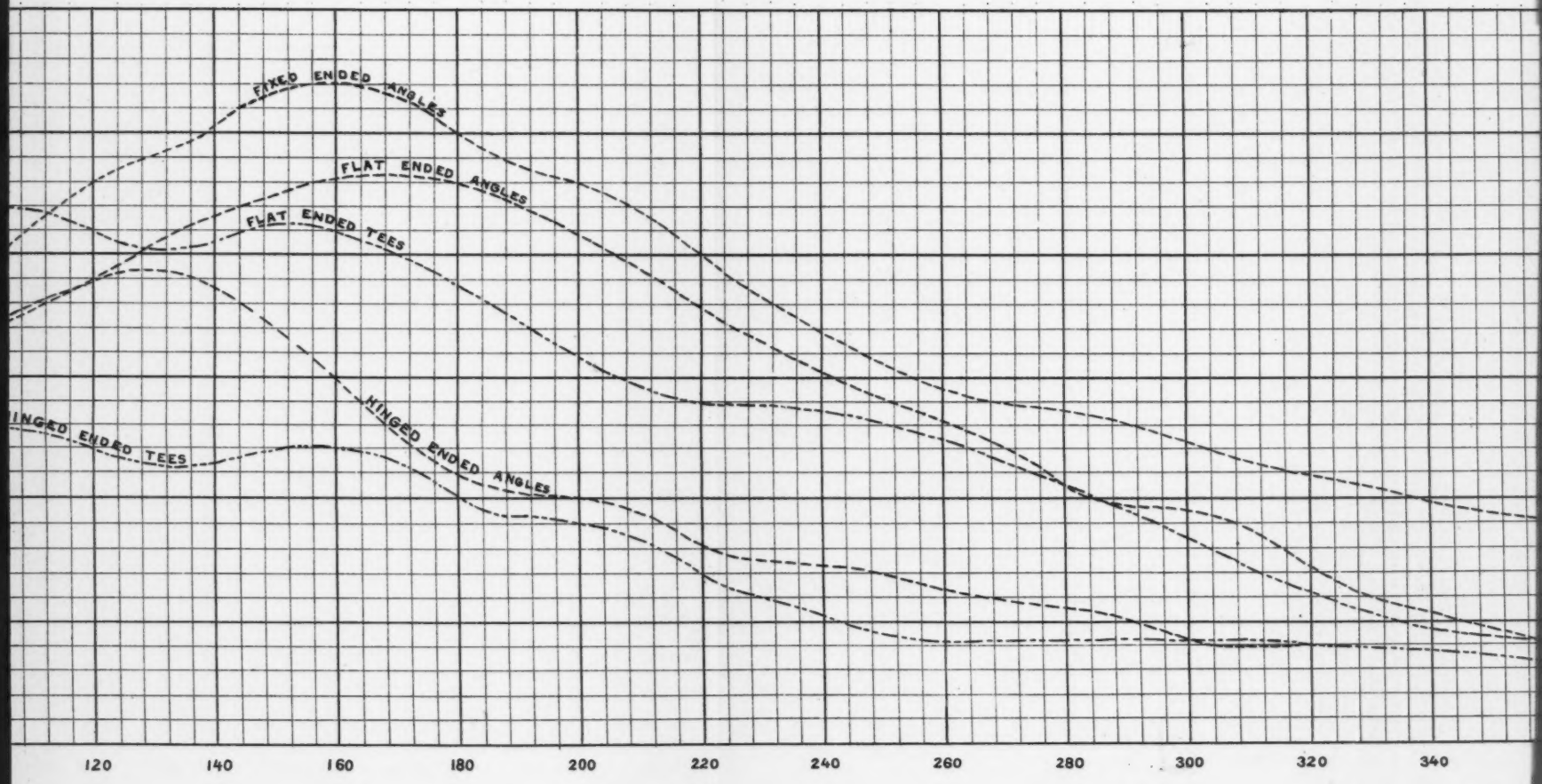
HINGED END


FIXED ENDED ANGLES
FLAT ENDED ANGLES
FLAT ENDED TEES
HINGED ENDED ANGLES
INGED ENDED TEES
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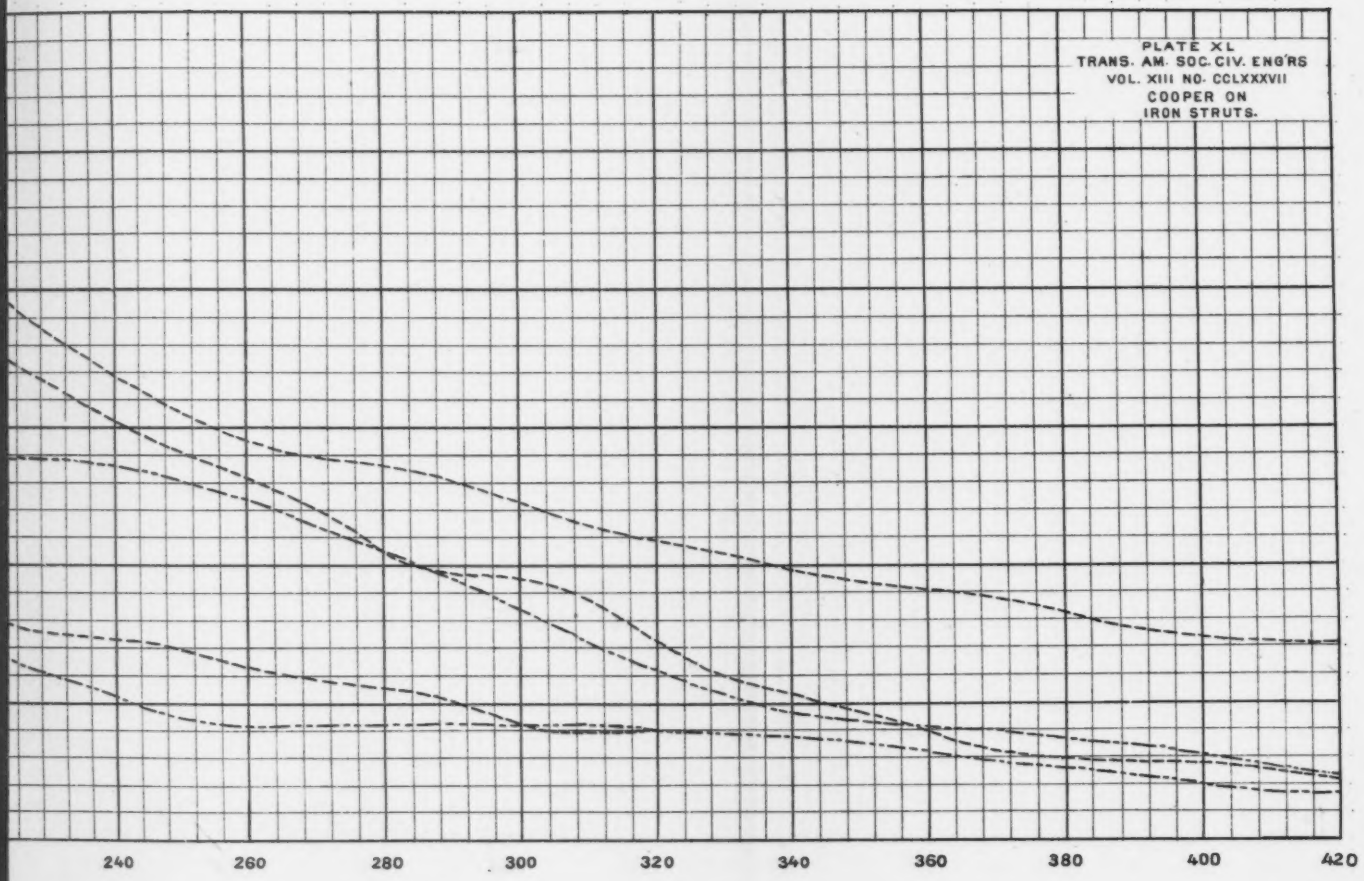
PLATE XL
TRANS. AM. SOC. CIV. ENG'RS
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COOPER ON
IRON STRUTS.
240
260
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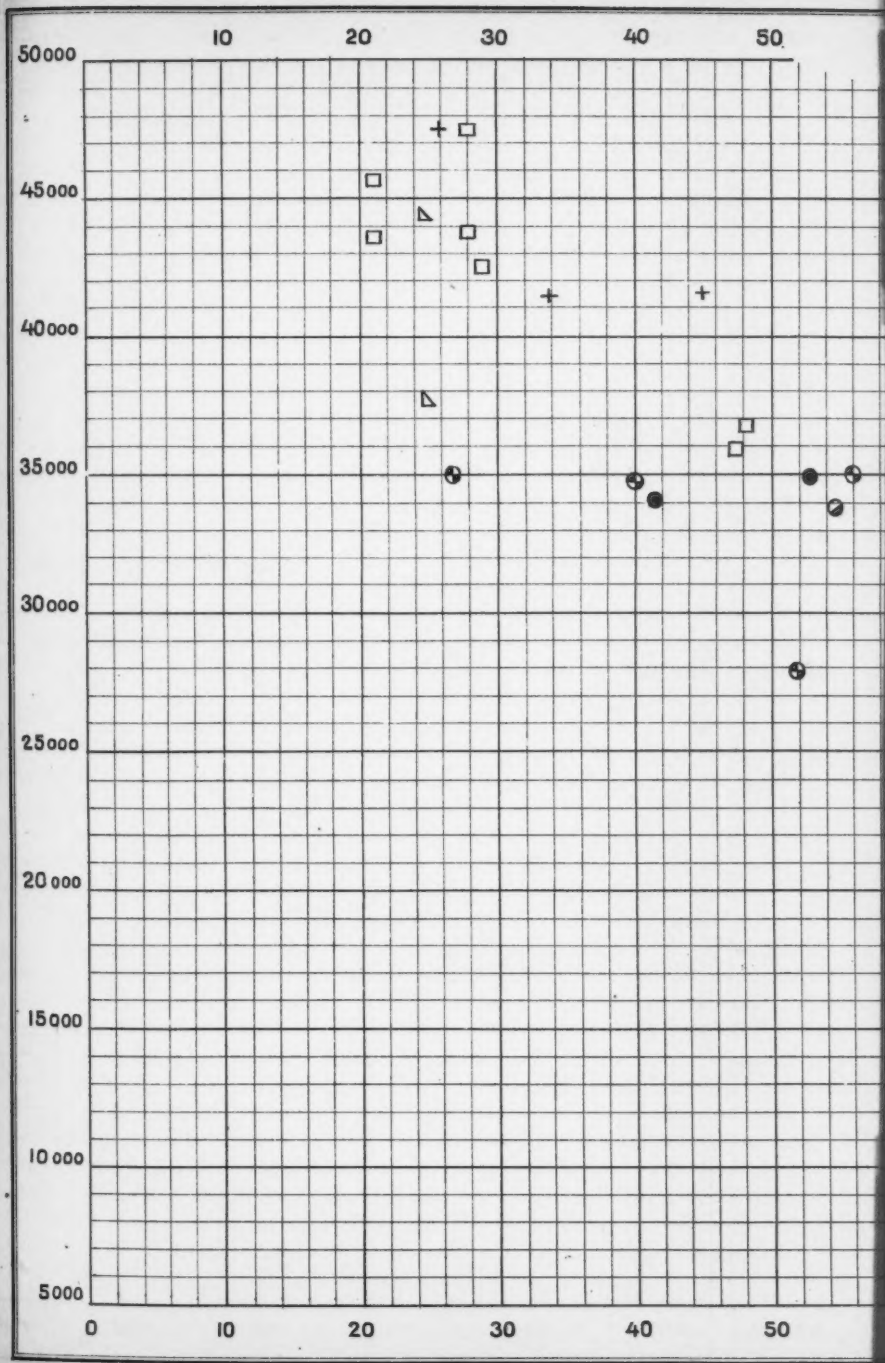
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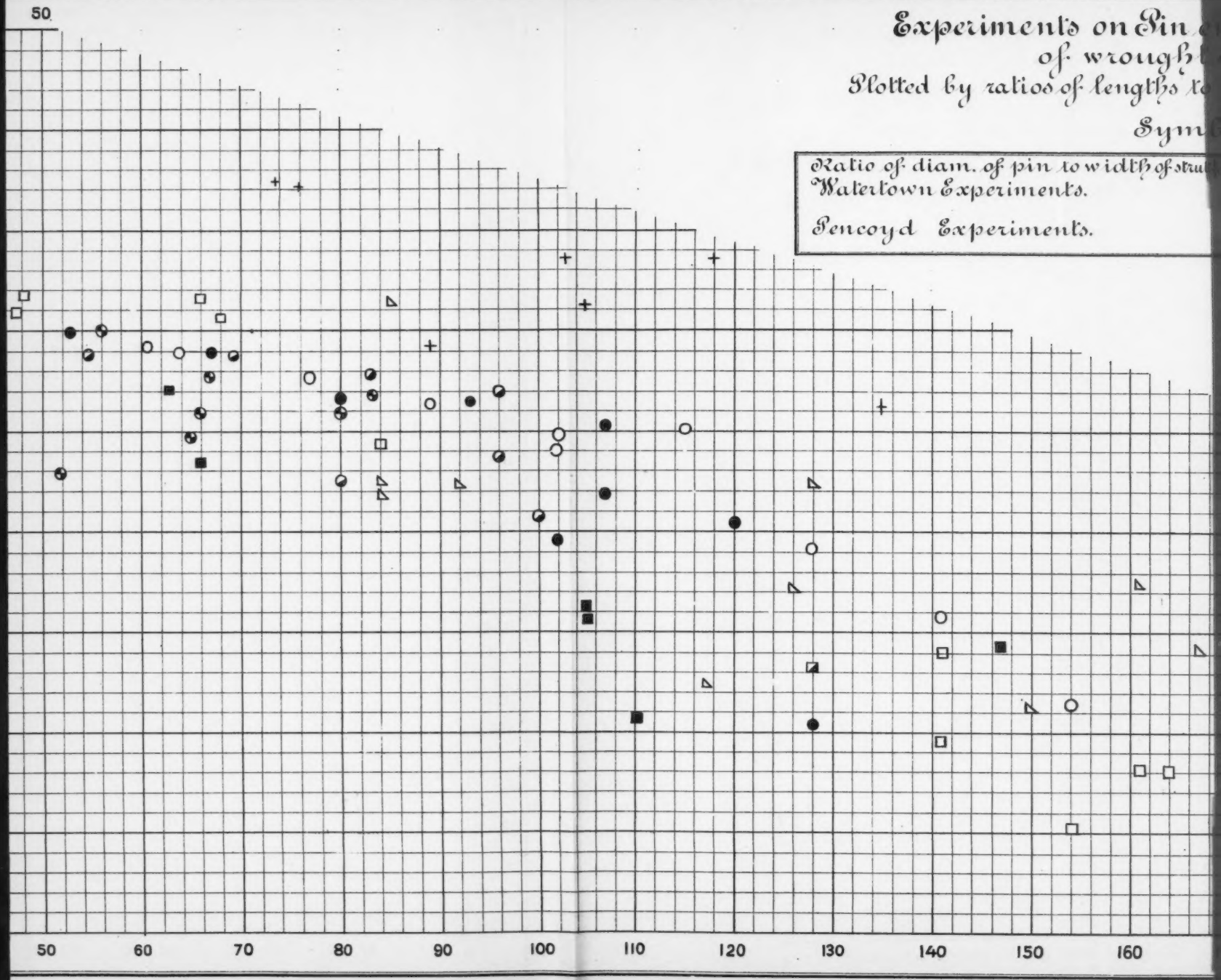
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Experiments on Pin
of wrough
Plotted by ratios of lengths to
Sym
Ratio of diam. of pin to width of stru
Watertown Experiments.
Pencoyd Experiments.
50
60
70
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90
100
110
120
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140
150
160

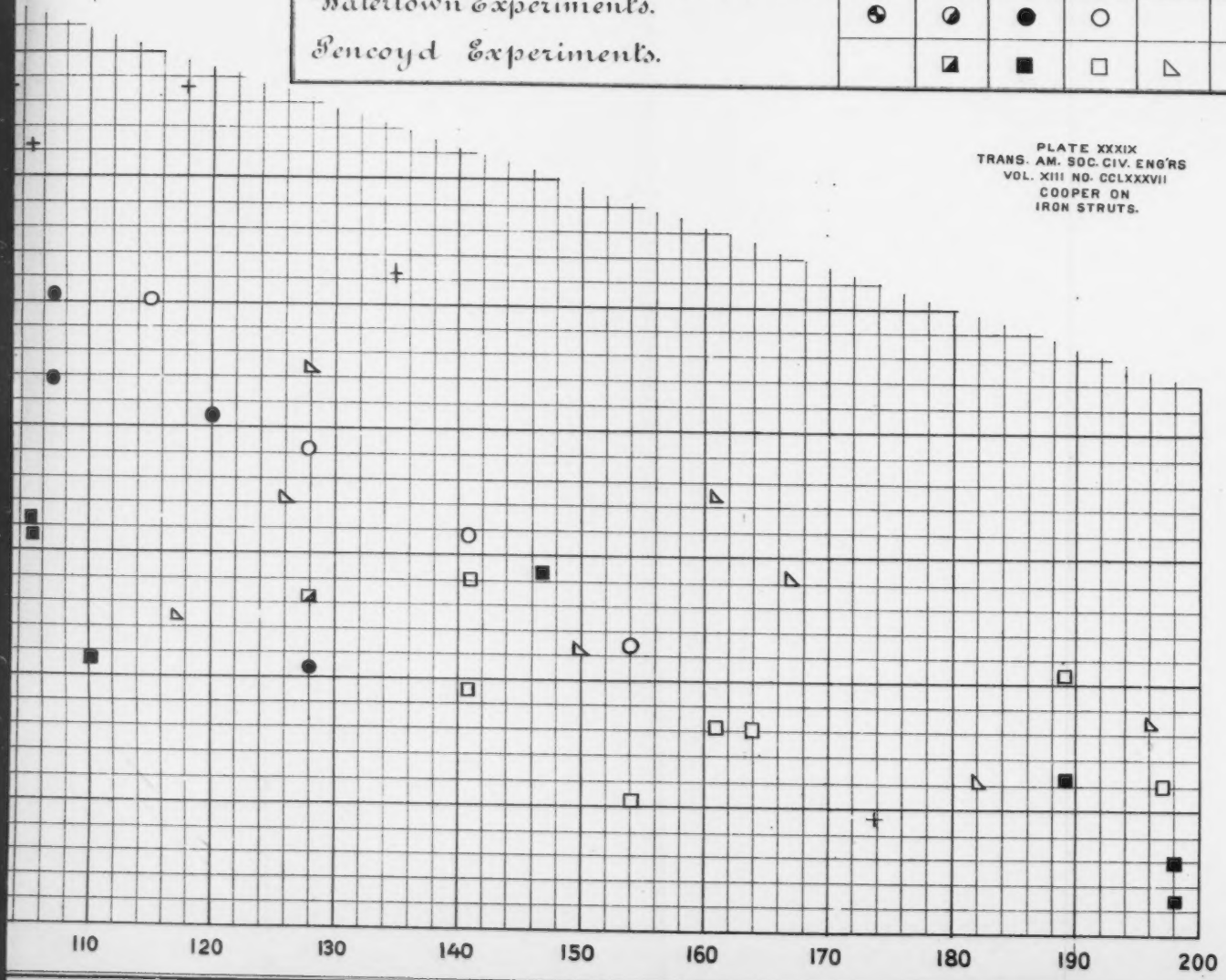
Experiments on Pin ended Columns of wrought Iron.

Plotted by ratios of lengths to least radii of gyration.

Symbols:

Ratio of diam. of pin to width of strut $\frac{1}{r}$ =	$\frac{25}{100}$ to $\frac{3}{10}$	$\frac{35}{100}$ to $\frac{4}{10}$	$\frac{45}{100}$ to $\frac{5}{10}$	$\frac{58}{100}$ to $\frac{67}{100}$	$\frac{9}{10}$	1.
Watertown Experiments.	◔	◑	●	○		
Pencoyd Experiments.		◪	■	□	◺	+

PLATE XXXIX
TRANS. AM. SOC. CIV. ENG'RS
VOL. XIII NO. CCLXXXVII
COOPER ON
IRON STRUTS.



in the material of the column, and the increased influence of possible side blows to columns under strain, I consider it very important, not only to keep our working strains within proper limits, but also to specify a limitation to the number of diameters to be used in all columns. In recent specifications I have placed such a limit at 45 diameters. This corresponds, approximately, to about 120 radii of gyration for the usual forms of bridge columns now generally used.

With this proviso, our practical formulæ could be reduced to very simple forms. The correct general formulæ for different forms of columns must necessarily be more or less complicated, to represent their strength for all possible ratios of length.

These, however, are only necessary in order to keep clear the law governing columns of long lengths, and to prevent false deductions being drawn by using *as general*, empirical formulæ only applicable to certain *limited* ratios. Within the above practical limits, we could adopt an immeasurable variety of equations, which with the proper constants would be equally true.

We refrain from proposing any new formulæ at the present time, as they would only be temporary ones, subject to further experiments, which we hope for, in the near future, upon full-sized bridge columns.

L. L. Buck, M. Am. Soc. C. E.—In the course of a number of tests of steel used in renewing the suspended superstructure of Niagara Railway Suspension Bridge, some were made that I have thought might be of interest in the discussion of the subject of "structural steel."

The tests in question were all made on the testing machine of Mr. W. L. Gill, in his car-wheel works, at Allegheny, Pa. The machine is a screw machine whose stresses are measured by a beam scale, and elongations by a micrometer reading to $\frac{1}{100\,000}$ inch. The screw admitted of stopping the machine at any time in the midst of a test, and that without its relaxing any portion of the stress that had been reached at time of stopping.

In all cases duplicate specimens were cut from steel plates. The specifications called for steel with an ultimate strength of 80 000 pounds per square inch, and a stretch of 15 per cent. in a length of 10 inches.

Among the specimens tested were some having as high as .48 per cent. of carbon, with an ultimate strength running from 95 000 to over 100 000 pounds per square inch, with a stretch of less than 8 per cent. in 10 inches.

It occurred to me that it might be well to test some of these higher specimens to ascertain their capacity for resisting shocks.

In order to make this test as severe as possible, the specimen was subjected to a stress about up to the elastic limit (as ascertained in testing the companion specimen), and while the stress was on, the piece was struck a smart blow with a hammer. None ever broke till after several blows had been struck. Afterwards some of the specimens were nicked across one side with a sharp cold-chisel, then subjected to a tensile stress nearly up to the elastic limit, and then struck with the hammer on the side opposite the nick. The first blow never broke the specimen, although a dozen specimens were tested in this way. Yet the same blow upon one of these hard specimens with the nick in one side, but no tensile stress, would break it short off.

This showed that, so far as severity was concerned, the experiments were failures. On the other hand, the tensile stress assisted the specimen to withstand the shock, simply by preventing vibrations and bending. It simply caused the stress due to the blow to be distributed over the section of the specimen, instead of being concentrated at the bottom of the nick, as it would be with no tensile stress on it.

The reason that it broke after several repetitions of the blow was, that each blow caused the specimen to slip slightly in the clamps at each end, thus reducing the stress till it was nearly all off when the piece would break.

Yet the tests possess some value.

They would at least seem to indicate that the high steel, if used in structures where the dead load considerably exceeded the live load (in the case of the tensive members), might be perfectly reliable, while in case the dead load was small and the live load large, in proportion, it would be dangerous. The constant stress produced by the great dead load would, as before explained, cause vibrations to be less, and to result only in slightly increasing the tension which must be distributed over the whole section.

The test by nicking was principally useful for the reason that it showed more conclusively the correctness of the reasoning. The nicked specimen with no tension upon it would break with a small blow, which would not break it when subjected to tension; while in the case of the specimen without the nick, it might withstand the blow, whether strained or not.

One of the questions which arises in the selection of the best grade of steel for a given purpose is the amount of work to be put upon it. If it is to be hammered very much while cold, it will become hardened more or less by the operation, and the drawing out of the hammered portion will subject the surrounding portions to considerable tensile stress. Again, it sometimes happens that in bars of steel there are hard places, producing, in some cases, internal stresses, although no work has been done on the bar. My experience has seemed to show that the effect of a blow in hardening was comparatively less in the higher steel than in the lower, and also that the higher grade was, if anything, less liable to have hard spots. By higher steel, I wish to be understood as meaning steel with .40 to .45 per cent. of carbon, and an ultimate tensile strength of 80 000 to 95 000, or even 100 000 pounds per square inch, of specimens whose cross-section is 1 to 1½ square inches area.

Steel with about .38 per cent. of carbon appeared to give about the most uniform results. None appeared to have excessively hard spots, while steel with .18 to .20 per cent. of carbon gave more trouble. In some cases a whole bar of the lower grade would be so hard as to break like glass by bending or twisting, and with no more appearance of permanent set in the vicinity of the fracture.

It was observed, in pulling a nicked specimen of the lower grade, without the hammer test, that the flaw at the bottom of the nick would appear and even open considerably before complete rupture, while in the higher grade mentioned the only evidence of stretching would be the dropping of scale from the surface of the specimen, but no perceptible opening of the nick till it popped open entirely.

WILLIAM H. BURR, Assoc. Am. Soc. C. E.—It is a matter. of congratulation among civil engineers that a subject of so much importance as the resistance of struts of angle iron and steel has received such careful and able treatment at the hands of Mr. Christie. And it is a matter of no less practical importance than physical interest to learn some of the characteristics of steel struts in this period of transition in structural material, at a time when undue conservatism has frequently prompted the assertion, that steel struts possess a resistance no greater and sometimes less than those of iron.

Now, while it is obviously true that we have only *begun* to learn the

characteristics of mild structural steel, and while it is equally true that these struts experienced, previous to testing, none of those manipulations in the shop requisite to the production of composite struts, and to which the higher structural steels have shown themselves at this time to be most delicately sensitive, it is certainly true that simple steel angle struts, in a state of nature so to speak, are very considerably stronger than similar iron ones, *i. e.*, in such lengths as are ordinarily used in bridge building.

As a matter of fact, these tests show that between the lengths of 30 and 120 radii of gyration mild steel struts (0.12 to 0.15 per cent. carbon) possess a resistance of about 6 000 pounds per square inch in excess of that of iron; this excess amounts to 15 to 20 per cent. of the resistance of the iron struts. Within the same limits the high structural steel (0.35 per cent. carbon) gave an excess of 2½ to 4 times as great.

These results, in connection with the successful shop manipulations to which mild structural steel is being almost daily subjected in some places in this country, seem to me to indicate with a high degree of certainty that such steel angle struts, in lattice bridge work, for example, will carry essentially the same excess of resistance over iron as shown by these experiments.

One matter brought out by Mr. Christie's experiments is of great practical importance in lattice bridge work. It is the fact that below 120 radii of gyration in length, the condition of ends is of little or no consequence, for the resistance is essentially the same whether the ends are hinged or fixed. It also indicates the possibility of a similar condition or limit for ordinary pin and flat end columns. It is not improbable that with the properly large pins now in use, the ordinary formulæ for pin-end columns give results somewhat less than the actual resistance.

One conclusion regarding the elasticity of structural steel of the milder grades reached by Mr. Christie, seems to me to be of doubtful character. On page 260, he regards it "certain" that steel shortens more than iron under compression. Table No. 5 shows such a result, but No. 2 does not. The latter appear to be the results of the tests of struts considerably below the elastic limit, while the former, I infer, were computed from observations on short pieces supported by sand in a tube. It is possible that the measurements in the latter case were faulty, though the extraordinary care and accuracy with which the experiments were made would render this improbable. Again, the co-efficient of

elasticity may have been taken in connection with the shorter pieces at or too near the elastic limit, a contingency which could not occur with the struts, except as a consequence of gross carelessness, which certainly did not exist. Any little incidental error would affect the results for a short piece much more than those for a long one. Kirkaldy did not find that the co-efficient of compressive elasticity for mild Fagersta steel was lower than that of iron. Allowing for the difference of character of the two steels, an unusually low compressive co-efficient still seems such an anomaly that doubts of its existence are reasonable.

One other result brought out by those tests is of great physical interest as well as of considerable practical value, as it furnishes a striking confirmation of the common theory of flexure. Mr. Christie has observed that above certain lengths the resistances of steel and iron angle struts coincide. Plate No. 8 shows that this limit is about 200 radii of gyration for mild steel and above 300 for hard steel, and beyond the limits of his experiments, though there can be little doubt of the existence of such a point.

The old formula for long column resistance first established by Euler is:

$$P = E \frac{a I}{l^2},$$

in which a is a constant, I the moment of inertia, l the length, and E the co-efficient of elasticity in bending. It is seen, therefore, that the only quantity entering this formula and dependent on the *character* of the material is E, since a depends only on end conditions. But the experiments of Mr. Christie showed that E is essentially the same for mild and hard steel and wrought iron. Hence, within the limits of application of Euler's formula, struts of those materials should possess the same resistance; or, rather, there should be a limit beyond which the resistances are the same. But the analytical conditions on which the theory of flexure is based, make it strictly applicable only to pieces whose lengths are very great in respect to their cross-dimensions, though this limit of length is indeterminate. The results of these tests confirm this indication of theory in a most satisfactory manner, and fix the limit for mild steel at 200 radii of gyration, while that for hard steel is probably not much above 300. Above these limits, column resistances are accurately represented by Euler's formula.

Tests of beams would possess more value and interest if different

lengths of the same cross-section were tested to failure, in order thus to discover the law connecting the modulus of rupture for bending with the length of span.

T. C. CLARKE, M. Am. Soc. C. E.—A paper may be so exhaustive that it leaves little room for discussion, and this may be said of the two papers of Mr. Christie on steel and iron. They are full of instruction, and the more such papers we get, the wiser we shall be.

It seems to me that such papers are more useful than the reports of Government Commissions.

No Government Commission can do as good work as private observers, because no half dozen men, even with a joint interest with the Ordnance Department in a Government testing machine, can know so well what is wanted, and so well how to experiment, as the whole body of scientific observers scattered all over the country.

No matter how bright five men may be in their own offices, put them in a board room, on a jury, or on a commission, they become soon affected with an obscure mental disease, not classified by physicians, which dulls their natural faculties, or else makes them quarrel violently. Then there is always one impracticable fellow, who will neither act himself nor let the others act.

The report is generally a compromise of opinions, of which every one is ashamed. Therefore, I say—more independent experiments and less Government Commissions.

When any of the great body of private observers has discovered any law or demonstrated any new set of facts, then is the time for the Government Boards, with their accurate methods, to come in and test the statement. If incorrect, let it be branded as such; if true, let it receive the Government stamp of approval, as the mint stamps its coins.

THEODORE COOPER, M. Am. Soc. C. E.—In regard to the idea, as advanced by Mr. Boller, of testing the same sized angles under many different lengths, I do not see that we would be any better off than now, unless it increased the number of tests. We are compelled to take general averages where the varying elements are so numerous. His method would simply give the tests of a particular angle, and would not represent the strength of other sized angles as closely as a general average

from a set of miscellaneous tests; such additional tests would, however, be valued as supplementing other tests.

HENRY R. TOWNE, M. Am. Soc. C. E.—As to the point made by Mr. Boller, that the experiments upon the flat-ended tees and angles indicate the hopelessness of arriving at any formula representing these results, I suggest that the difficulty may be, not with the formula, but with the materials. In other words, are not the discrepancies due to variations in material? Experience has shown this to be one of the factors in such cases—that material cut from the same bar gives different results under test. That accounts for much of the variation in the experiments, and it does not follow that good results may not be accomplished by such experiments, and, as they are increased, a correct average may be deduced from them. One lesson to be drawn is the need of greater effort on the part of manufacturers of iron to give us good materials to work with.

Mr. THEODORE COOPER.—I would like to ask Mr. Christie whether he tested the exact parallelism of the bearings of the fixed-ended struts while in the machine. If in fixed-ended struts, the anomalous results of the short lengths could not be due to this want of parallelism.

JAMES CHRISTIE, M. Am. Soc. C. E.—I would answer that in all cases the body of the specimens was practically square with the ends. We may have put in the machine pieces whose ends were not exactly square with the axis, or not parallel with each other, but the operator regulated the machine until the abutting plates came flat on the ends of the specimens. In the fixed-ended struts the same precautions were taken, but probably the reason why the short fixed-ended struts failed apparently lower than flat ends was due to the short lengths necessitated by the limited capacity of the testing machine.

We measured the length between the clamps, whereas the effective length may have reached some point within the clamps; such an addition to the recorded length would make an appreciable difference in results in the case of such short specimens.

Mr. HENRY R. TOWNE.—In making the transverse tests, how were the specimens supported?

Mr. JAMES CHRISTIE.—They were supported on a beam with two tees

with rounded edges, which were lubricated so that the specimen beam could slide.

E. B. DORSEY, M. Am. Soc. C. E.—I am now making some experiments on mild steel under compression, which prove its superiority under compression as well as tension. When they are completed I will give the results to the Society in a future paper.

Mr. THEODORE COOPER.—In the comparative diagram of Mr. Christie of the ultimate resistances of steel and iron, which is a subject of considerable interest, we see that from about 40 up to 120 gyrations the mild steel averages from 15 to 20 per cent. in compressive strength above that of iron. Milder steel, I think, would be still less. Hard steel I do not think any practical bridge engineer would recommend as suitable for built compressive members. With this new material, of which our experience is practically nothing, when compared with iron, the best gain that we can expect is 20 per cent. With a gain as slight as that, a cautious man would be careful before putting in a new material, which is so much more readily affected by the processes of manufacture and transportation. I reiterate what I said before: let us go very cautiously into the use of steel until we can have more extensive knowledge of the subject. In connection with that remark, I would like to add that in our great structures, which are usually skeleton structures, we also depend upon uniformity in the action of the different parts. If I understand Mr. Christie correctly, steel varies very much in its action. It would, therefore, seem to be more difficult to make the different members act in harmony with each other.

Mr. E. B. DORSEY.—Did you not find in your experiments that mild steel gave much more uniform results than iron in tension and compression, and was not the elastic limit much more uniform also?

The tests that I have seen made on English mild steel gave much more uniform results than those on iron; and these are sustained by verbal statements made to me by the large ship-builders on the Clyde, and other large fabricators of iron and steel.

Mr. JAMES CHRISTIE.—It is difficult to answer that question until we can make a sufficient number of tests. We found considerable variation in the elasticity of steel. I would state here that, at first, in our experiments, the steel seemed more uniform in elasticity than iron; but con-

tinuing the experiments, that did not seem to be the case. Especially in compression, the elasticity of steel was fully as variable as that of iron.

Mr. E. B. DORSEY.—Were the tests made on Bessemer steel?

Mr. JAMES CHRISTIE.—Yes, sir.

Mr. E. B. DORSEY.—In England the preference for structural purposes, including ship-building, is in favor of the Siemens-Martin or open-hearth steel over the Bessemer, on the ground that it is much more regular and uniform in its character. As an evidence of this, the price is higher—about fifty per cent. greater.

Mr. JAMES CHRISTIE.—An establishment in Pittsburgh, extensive producers of structural steel, built an open-hearth plant some years ago, which was afterwards destroyed by fire. They rebuilt with the Bessemer converter, and now claim that their choice was a wise one, not only considering economy, but also uniformity of product.

Mr. THEODORE COOPER.—In a conversation with one of the leading manufacturers of steel in Pittsburgh on the subject of Bessemer and open-hearth steel, he told me that he had advocated the open-hearth, but after a series of careful experiments he had to acknowledge his error against his previous conviction, and found that there was more uniformity in the Bessemer than could possibly be by the open-hearth. He did not mean by that, that the mild steel that is in the market now is uniform. It can be made so, but is not. The mild steel now being sold varies from material as hard as glass to material as soft as lead, not only in different bars, but in the same bar. I have known of steel angles, while being punched (which is a good test of uniformity), that would commence at the end and punch very softly, then, after 3 or 4 feet, the punching would go off like a torpedo. It then would again strike soft places, sometimes three or four times in one bar. It is not the fault of the material as much as want of care in securing uniformity. I believe, until structural steel is more carefully manufactured, it is not safe to use it under such high strains as has been done.

EDWARD P. NORTH, M. Am. Soc. C. E.—I would like to ask the relative cost of Bessemer and open-hearth steel. I believe there is a decided difference. Can any gentleman here tell me?

Mr. E. B. Dorsey.—In England the Bessemer is principally used for rails, and sells now for £5 per ton; and the open-hearth made by the Siemens-Martin process sells for £8 per ton. This latter is used entirely for structural purposes. I do not think the difference in the cost of manufacture amounts to £3, or $15, per ton—probably not half of it.

Mr. Theodore Cooper.—Much time and money has been expended to develop a perfect steel rail, but manufacturers in this country have paid little attention to the development of structural material.

C. C. Schneider, M. Am. Soc. C. E.—While I admit the difficulty of obtaining steel of uniform quality from our manufacturers, I am of the opinion that a fair degree of uniformity can be secured by rigid inspection.

The specifications should be strictly adhered to, and all material which fails to come up to the specifications in any one point should be promptly rejected. In this way a material of fair uniformity may be obtained.

According to my experience, open-hearth steel can be more relied upon than Bessemer steel.

Percival Roberts, Jr., M. Am. Soc. C. E.—In regard to open-hearth and Bessemer steel, the cost of manufacture is probably not very different. The great trouble, both here and abroad, is that, as a rule, the Bessemer is used principally for rails, whereas the open-hearth is a low carbon steel and used for different purposes. When low carbon steel is manufactured the cost may be larger, because the pig iron used for rail steel would not do for a low carbon steel. I think that our experience with Bessemer steel has been as satisfactory as with open-hearth, provided we got it from a Bessemer works which understood making the low grades of steel. I do not think the cost of manufacture is very different in either process. If we should ask Mr. John Fritz he will probably prefer the Bessemer. But Mr. Wellman will prefer the open-hearth. If the specifications define what is required you can probably get suitable material by either process.

When contemplating making these tests we corresponded with the authorities at the Watertown Arsenal, and had prepared about twenty tons of specimens, but in the end found no appropriations for tests. We then concluded to make the tests which Mr. Christie has made, and

I do not think that our experience has been that steel has any more homogeneity than iron. Taking steel of the same chemical analysis, the physical tests will frequently vary very much. We have no prejudice against steel. We would like to roll it, but do not see that we get any more homogeneous results than with wrought iron.

As to the superiority of steel in ship-building, it comes from the use of bad iron. I recollect a few weeks ago being in a ship-yard where they used steel for boilers. A plate 14 feet in length, ⅜ inch thick, and 5 feet in width, was flanged with semicircles. They punched the plate, laid it down, and the following morning they found cracks in it 2½ feet in length. It only tends to show that when using steel in structures careful supervision and inspection is necessary. It will not bear the same treatment as iron, and requires different methods of manufacture. I do not know whether Mr. Christie remembers some tests made on steel slightly nicked showing its behavior under the drop.

Mr. JAMES CHRISTIE.—The experiments to which Mr. Roberts alludes were for determining the difference between steel and iron in resistance to impact. As these experiments were not concluded, they are not mentioned in my paper. Several round bars of iron and steel, both mild and hard, were subjected to the ordinary test for railroad axles; that is, placed on supports 3 feet apart, and struck in the middle with a drop weighing 1 600 pounds.

When the surfaces of the materials are not disturbed, the steel will usually endure much severer treatment than iron. But when the surfaces are nicked or scratched, the action is contrary.

This was true for the very mildest steel, as well as for rail steel. As an illustration of the subject, I have the records of the endurance under impact of 3-inch round bars of mild steel and of iron. Both bars were scratched around their bodies, just sufficient to remove the scale and expose the bright metal. The depth of the cut, which was made with a saw file, was not measurable. The bars were turned after each blow so as to reverse the deflection. The steel bar broke under the fourth blow, the weight falling 3 feet. The iron bar stood 12 blows, and broke under the 13th blow, when the weight was dropped 4 feet.

Mr. HENRY R. TOWNE.—Mr. Christie has mentioned the fact of these specimens breaking at the point where they were slightly nicked. I would mention that in our testing machine we have found it necessary

to mark one side of the specimen, more for locating elongation than to determine the points where breakage would take place. A very small nick, particularly on the corners, will define itself.

Mr. THEODORE COOPER.—There is another interesting point in steel; that is the ease with which it can be cracked and the crack still be invisible. In other words, the ocular inspection of steel is different from iron. You can examine an iron bar, and feel that the iron is in a condition to be used; but in the same kind of steel bar, 50 per cent. may be in a condition of rupture and not be discovered. In the St. Louis Bridge I had a member put in the machine. It stood all the tests. I marked it accepted, and as the men were swinging it around I saw a thread of light across the bar. I stopped the men and examined the bar as carefully as I could, and could not find it until I took a magnifying glass, and found the bar cracked three-quarters way across. I then had a man strike it a blow with a sledge, and it split in half. Steel is a crystalline metal; iron is not. It is impossible to have a piece of iron cracked so badly as to be dangerous without detection when examined carefully.

Mr. E. B. DORSEY.—What was the tensile strength?

Mr. THEODORE COOPER.—The tensile strength was 120 000. The point Mr. Christie has made is very important. For boiler construction and that sort of thing, steel is safe, but for bridges it is not, and a material that a scale or mark weakens is not safe to use.

Mr. E. B. DORSEY.—I am not astonished at the trouble Mr. Cooper experienced in the use of steel in the St. Louis Bridge, but I am very much astonished that they succeeded in building the bridge at all, and that it stands after being built, when it is considered that the steel of which it is built has a tensile strength of 120 000 pounds per square inch, being fully 50 per cent. greater than is considered safe by the boldest engineers now, and 70 per cent. greater than I would advocate. I do not think steel should ever be used of over 70 000 pounds tensile strength per square inch.

Mr. THEODORE COOPER.—Mr. Dorsey asked me what was the tensile strength of the steel stave, which I mentioned as showing the crack after passing inspection. He appears to conclude that the defects I have

mentioned as peculiar to steel were due solely to steel of this high tensile resistance. He is wrong. While I agree fully with him in the belief that 70 000 pounds steel is as high as we should use for tensile resistances, we cannot shut our eyes to the fact that steel of all grades, even the softest, is essentially crystalline in structure. It therefore has, to a more or less extent, the defects due to this crystalline character, among which are the increased difficulties of ocular inspection and the injury to its resistance under impacts or sudden strain from changes of sections, by rivet holes, nicks, or even slight scratches. That these defects are less in the softer steels is undoubtedly true. But even in these softer steels they ought not to be overlooked or ignored.

As I have so often stated before, we are past the age when purely statical tests can be considered as the sole criterion of the capabilities of any structural material. We need more impact tests, and upon such tests only can we form a true comparison of the relative merits of structural material for bridge purposes. If manufacturers will put upon steel members 50 per cent. more strain than is allowed upon similar members of iron, simply because it is steel, we have a perfect right to demand of them their proofs.

It has not yet been shown by any recorded tests that steel columns made of mild steel will stand over 20 per cent. higher than iron ones.

Is an allowance of even this much above the allowed strain on iron columns justifiable, considering the decreased resistance of steel under sudden strains or impacts, when nicked or even slightly scratched ?

Mr. E. B. Dorsey.—It is true that mild steel, or steel of 70 000 pounds or less tensile strength per square inch, is crystalline in structure, but from the testimony of the numerous workers in it, that I have interviewed, I do not think that it has any of the bad qualities ascribed to it by Mr. Cooper because it is crystalline in structure, like the harder or stronger steel to which Mr. Cooper compares it. On the contrary, all the workers in mild steel that I have seen prefer it to iron, on account of its being easier worked and with much less loss.

William J. McAlpine, Past President Am. Soc. C. E.—Can you tell us what proportion of carbon enters into the composition of what is termed in the paper just read "mild steel" ?

Mr. James Christie.—In answer to that question I will say, that while we can draw no distinct boundary line between mild and hard

steel, yet when we speak of mild steel, it is generally assumed that the carbon ratio is not over $\frac{15}{100}$ of 1 per cent., because steels lower in carbon than this will usually endure chilling in water from a red heat, and be capable afterwards of being bent without cracking; whereas, when the percentage of carbon equals or exceeds $\frac{15}{100}$ of 1 per cent., the specimen, if submitted to such treatment, will not endure much subsequent bending without fracture.

Mr. Dorsey states that in Great Britain it is generally assumed that soft steel and iron will be on a uniform commercial basis when the former sells at 2 cents and the latter material at 1½ cents per pound. This statement assumes an average efficiency for steel over iron of 33 per cent.

It is somewhat hazardous to say to what extent steel is better than iron, without considering the particular purpose to which it is to be applied. The mild steel most frequently used for ship and bridge building contains about .12 carbon, having an ultimate tenacity of 65 000 and elastic limit of 39 000 pounds per square inch of cross section. This steel, when used to resist tensile or transverse stress, will have an advantage over iron of about 30 per cent. But steel beams will, on an average, deflect under given loads as much, or nearly as much, as iron beams of the same dimensions. Here the question of efficiency may not be one of absolute strength, but one of stiffness. Again, in the form of struts and columns, when used in forms whose length as compared with cross section is a minimum, the steel will possess the same advantage of 30 per cent. over iron. But this superior resistance rapidly diminishes as the length of the strut is increased. Our experiments show that when the ratio of length over least radius of gyration is 150, the mild steel strut exceeds the iron strut in resistance about 10 per cent.; and that when this ratio is 200, practical equality between the two metals is obtained. Hence, we cannot make any general comparison as to the relative efficiencies of steel and iron, without considering their application.

Mr. E. B. Dorsey.—The prices named applied only to ship-building, and showed the relative proportion that the ship-owner could afford to pay for iron or steel ships.

A ship built of iron at 1½ cents per pound, would cost less than one of steel at 2 cents per pound, both of the same gross tonnage, but the net carrying and earning capacity of the latter would be greater,

owing to having less dead weight, caused by being built of a stronger material, consequently less of it; this allows the ship-owner to make the difference of 33⅓ per cent. of steel over iron.

In ordinary structural work it will not be safe, in my judgment, to assume the efficiency more than 20 to 25 per cent.

Mr. J. F. Holloway.—I would like to ask whether you have tested specimens cut from different parts of beams, to observe the change effected by reduction of section?

Mr. James Christie.—We have tested many such specimens, both in tension and compression, and while undoubtedly the more the steel is condensed by the rolling process the higher its resistance will be, yet we have never observed the wide differences found by Kirkaldy in his experiments on Swedish steel. The most striking difference between steel and iron seems to be in the greater variation of elasticity under compression. In the former material, in tension steel is the stiffest and probably most uniform, while in compression it is reverse as compared with iron. The specimens exhibited here illustrate this: they were cut from a 12-inch I beam, which was tested transversely in a 13-foot length. In tension the ultimate strength was 70 000 pounds, and elastic limit 42 000 pounds per square inch, and modulus of elasticity 32 800 000 pounds. In compression the elastic limit was 38 000 pounds per square inch, and modulus of elasticity 17 500 000 pounds. Transversely, the modulus of elastic resistance was 39 000, and the modulus of elasticity 28 000 000 pounds. The chemical analysis of this steel showed .15 carbon.

Mr. Percival Roberts, Jr.—I desire to say, in regard to these experiments, that while we have no prejudices in the matter, yet they certainly did not give the results we expected to get from steel. The experiments were intended to be made a year or eighteen months ago; we were met on all sides by engineers saying that it was only a matter of a few years before we should have steel entirely. At the same time, no one was able to give us any data in regard to how much better results could be obtained from steel than from iron. It resolved itself into a very serious question, whether we should be compelled to go entirely into the manufacture of structural steel instead of wrought iron; and being unable to get any information whatever, we determined to make these experiments for our own use. And after making them and getting the results we did, we

concluded that for a few years to come, at least, we might as well keep on manufacturing wrought iron. We frequently hear statements similar to Mr. Dorsey's of this afternoon on this subject. They are always general in their character. They say steel is stronger than iron. We all know that; but at the same time they furnish us no definite information on which to base calculations. I think that steel will not come into general use until we come to the conclusion that when we use it we are not working with wrought iron. Steel is a different material, and must be treated in a different manner, just as you cannot treat cast iron the same as wrought iron. It is a crystalline material, entirely different in its nature.

Mr. WILLIAM J. MCALPINE.—Will you give us a definition of wrought iron, steel and cast iron?

Mr. PERCIVAL ROBERTS, Jr.—A gentleman said this morning that anything that could not be said in ten minutes could not be remembered, so I will not undertake it. I do not think it would be possible in a few words to define the difference. I do not know of any better definition of steel than that it is a crystalline metal, produced by fusion, and capable of being wrought at a red heat.

O. E. MICHAELIS, M. Am. Soc. C. E.—Alluding to Mr. Roberts' statement in regard to the difficulties in the mechanical manipulation of steel, I understand that it is claimed that in the open-hearth steel they can obtain constant results, as regards its qualities, by mechanical treatment. I would like to hear from some gentleman present on that subject.

Mr. PERCIVAL ROBERTS, Jr.—I do not think that, so far as we know, there is any difference in quality between the open-hearth and Bessemer steel. Equally good material can be made by either process; but in handling large masses, such as large castings of a number of tons weight, the open-hearth process is the better. You can handle larger masses of material by the open-hearth process. In our experiments on both classes of material, we have had equally good results from each—we saw very little difference. I will say that authorities do not agree on the subject. Those who have Bessemer plants consider Bessemer steel the better, and those who have the open-hearth are in favor of that.

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TRANSACTIONS.

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CCLXXXVIII.
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EXPERIMENTS ON THE HUMPHREY TURBINE WATER-WHEEL, AT THE TREMONT AND SUFFOLK MILLS, IN LOWELL, MASS.

By JAMES B. FRANCIS, Past President Am. Soc. C. E.

READ AT THE ANNUAL CONVENTION, JUNE 10TH, 1884.

In December, 1880, the Humphrey Machine Company, of Keene, N. H., made a contract with the Tremont and Suffolk Mills, one of the manufacturing corporations of Lowell, Mass., to furnish and set up complete in their mills, a turbine water-wheel of about 275 horse power, when used under a working head of 13 feet, to replace a Fourneyron turbine of about 162 horse power, under the same fall; the compensation for the same to depend on its performance under a test to be made under the supervision and direction of the writer.

The wheel is of the class combining an inward flow with a downward and outward discharge. It has some peculiarities of form and construction, the invention of Mr. John Humphrey, of Keene, N. H., for which he was granted letters patent No. 260 202, dated June 27th, 1882.

It is designed to run generally to its full capacity, that is, with the regulating-gate fully open, and in connection with a steam engine and other water-wheels, and is not provided with a governor, the speed of the mill being regulated at the steam engine or other water-wheels.

The turbine experimented on, together with the apparatus for weighing the power developed and the quantity of water discharged, is represented on Plates XLI and XLII.

Plate XLI, Fig. 1, is a vertical section through the centre of the penstock, wheel, weir, &c. Fig. 2 is a horizontal section on a larger scale, showing part of the stationary cylinder, regulating-gate, and two of the guides. Plate XLII is a plan of the entire apparatus. *AA*, the wheel, carrying thirteen buckets, *a a a*. *B B*, a stationary, cylindrical casing, which incloses the wheel and supports other parts of the apparatus. It has twelve orifices, *q*, in it for admitting the water to the wheel. *CC*, the regulating-gate and chute-rim. It consists of two curved crowns, *v v*, between which are twelve guide-vanes, *s s*, corresponding to the apertures in the stationary casing *B*. The regulating-gate is caused to rotate between fixed stops, so placed that when the gate is in contact with one set of stops, the orifices *q* in the fixed cylinder *BB* are closed, and no water is admitted to the wheel; and when the gate is rotated in the opposite direction, and is in contact with the other set of stops, the orifices in the cylinder *BB* are fully open, as shown in Fig. 2, Plate XLI. The regulating-gate is caused to rotate by means of the pinions *DDDD*, working into racks on the periphery of the gate, and operated by the hand-wheel *G*, through the wheel and pinion *F* and the worms *EE*.

The gate rests on the balls, *b b*, which are designed to revolve with the movement of the gate. The buckets, *a a a*, are of bronze, with warped surfaces on which the water impinges. They are fastened to the cast-iron centre-piece by flanges, forming parts of the buckets, which fill up the spaces between the buckets and form a bronze lining to the centre-piece, to which they are fastened by bolts with counter-sunk heads.

The weight of the wheel, upright shaft and friction pulley is supported on the bearing at the top of the shaft, which has a series of necks and projections to increase the bearing surface. There is also a wooden step, 9 inches in diameter, at the bottom of the shaft, to support the same when the top bearing is taken off or fails in any way.

The water is conducted from the forebay *R* to the wheel by the pen-

stock *TT*. The wrought-iron part, connecting with the forebay, formerly supplied the Fourneyron turbine, and is somewhat too small for this wheel, causing too great a velocity and a corresponding loss of head, amounting to about 4 inches when the regulating-gate is fully open. The connection of the wrought-iron part with the cast-iron should have been on one side instead of central, in order to give the water immediately over the wheel a rotatory motion in the direction of the motion of the wheel, which would prevent part of the loss of head due to the velocity of the water in passing from the forebay to the wheel. In these experiments, the height of the water above the wheel being taken immediately over the wheel, the loss arising from the insufficient size of the penstock is eliminated.

The apparatus for weighing the power developed by the wheel, or the useful effect, was arranged by Mr. John Humphrey.

The main gear for transmitting the power from the wheel was removed and the friction pulley *HH* substituted. It is of cast-iron, keyed to the vertical shaft, and has a series of channels cast in its rim, through which water circulates to carry off the heat developed by the friction of the brake. *II* is the brake, which is connected with the bell-crank *M* by the link *K*, operating on the knife-edges *L L'*. The bell-crank *M M* is suspended on the knife-edge *N* and carries the scale-pan *l*. The bell-crank and scale-pan were balanced by the weight *P*, by means of the beam *OO*. *Q*, the hydraulic regulator, consisting of a piston moving freely in water, the piston being about a sixteenth of an inch less in diameter than the cylinder, allowing the piston, and with it the bell-crank and brake, to move slowly through a limited range without sensible resistance, the water on one side of the piston passing to the other side through the narrow space between the edge of the piston (which is a plate of iron about half an inch in thickness) and the cylinder. Without the hydraulic regulator, or its equivalent, there is some uncertainty in the indications of the Prony brake, from its sudden movements and violent shocks, apparently caused by variations in the friction due to the expansion and contraction of the friction pulley from changes of temperature, from variations in the lubrication, and perhaps from other causes. The hydraulic regulator compels a slow movement of the arm of the brake, but allows it to yield readily to a preponderating force in either direction.

The brake lever *II* is of wood. The plate iron strap *k k* is lined with

a series of wooden blocks *n n*, which cover about two-thirds of the periphery of the friction pulley, and is tightened up by a screw operated by the hand-wheel and pinion *o*. The necessary tension, in order that the friction may be equal to the effect of the weight in the scale-pan, is adjusted by the operator of the hand-wheel, who keeps his eye on the zero of the gauge *u* and the pointer *t*; if he sees the pointer to be above the zero, he slackens the screw, and if below he tightens it.

R is the surface of the water in the forebay, *S* the surface of the water in the wheelpit; the difference of level between the two surfaces being the actual fall of the water. The water in the wheelpit being more or less agitated, to obtain its mean height, a pipe, *c c*, was laid on the wheelpit floor, connecting at each end with a water-tight box *d*, containing the wheelpit gauge. The pipe was 1¼ inches in diameter, and had 32 holes $\frac{3}{16}$ inch in diameter on its upper side, varying from 2 to 2½ feet apart, the spaces between them being the greatest near the box.

In the contract under which the wheel was furnished, and its power guaranteed, it was provided that the "working head" was to be taken, which was construed to mean the effective head at the wheel, which was ascertained by means of the pipe *e e e* connected with the penstock directly over the wheel. This pipe terminated at a glass tube placed opposite a gauge *f*, at which the height of the water above the wheel was observed; and the working head, or "total fall acting upon the wheel," used in computing the results of the tests, is the difference in the heights of the water at this gauge, and at the wheelpit gauge in the box *d*.

The quantity of the water discharged by the wheel was gauged at the weirs *U U*, both weirs being of the full width of the channels leading to them, that is, there was no contraction at the ends. The length of the easterly weir was 11.22 feet, and of the westerly weir 10.98 feet. To prevent the agitation of the water in the wheelpit from affecting the discharge over the weirs, racks or gratings, *V V*, were placed at the outlets of the wheelpit, causing a sensible fall in passing through them, and a nearly uniform flow over the whole section of the channels, and preventing eddies and other irregularities in the flow toward the weirs.

The height of the water on the down-stream side of the weir was regulated so as to be sufficiently below the crest not sensibly to affect the flow over the weir. Experiments made in 1848, and reported in Lowell Hydraulic Experiments, indicated that when the height of the water

on the down-stream side of the weir was 0.23 feet below the crest, the depth on the weir being 0.85 feet, there was no sensible obstruction to the flow. In some experiments made on these weirs, subsequent to the experiments on the power of the wheel, and reported in the following paper, it was found that when the height on the down-stream side of the weirs was 0.27 feet below the crests, the depth on the weirs being about 2 feet, there was no sensible obstruction to the flow. In all of the turbine experiments given in this paper, the water on the down-stream side of the weirs was not less than 0.30 feet below the crests, and it is assumed that there was no sensible obstruction to the flow caused by the water on the down-stream side of the weirs.

The quantity of water flowing over the weirs has been computed by the formula given in Lowell Hydraulic Experiments, viz.:

$$Q = 3.33\ LH^{\frac{3}{2}}$$

corrected for the velocity of the water approaching the weir by the method described at page 252 of the third and fourth editions of the same.

The depths on the weirs were measured by the hook gauges XY, placed in water-tight vessels connected with the channels of approach to the weirs by the pipes ggg, 1 inch in diameter, the parts of the pipes in the channels of approach, parallel to and 6 feet from the weirs, being enlarged to 1.5 inch diameter, and perforated with 12 holes $\frac{3}{16}$ inch diameter for each weir.

m is a pipe, provided with a valve, through which the water in the wheelpit could be drawn off when the water in the raceway on the down-stream side of the weir was sufficiently lowered; this valve was closed during the experiments on the turbine.

Experiments were made to ascertain the leakage during the experiments from the wheelpit, and through the floor and sides of the woodwork of the weirs, and their connection with the wheelpit; it was found to be very small, not exceeding 0.01 cubic foot per second.

The water used for cooling and lubricating the friction pulley was gauged in a water-meter, and found to be about 0.03 cubic foot per second; this water passed into the wheelpit, and is included in the quantity gauged at the weirs. The difference between this quantity and the leakage from the wheelpit is taken at 0.02 cubic foot per second, and this quantity has been deducted from the quantity gauged at the

weirs to give the quantity discharged by the turbine in each experiment.

The power developed by the wheel was measured by the Prony brake, the friction pulley *HH* of which had a rubbing surface 5.44 feet in diameter and 2.70 feet wide. Its weight was about 10 300 pounds, exceeding the weight of the crown gear, which it replaced for the purpose of these experiments, by about 5 800 pounds. The brake was suspended from the floor above by a perpendicular rod attached to a bolt passing through its centre of gravity, the length of the rod being adjusted by a turnbuckle, so that the brake just cleared the flange at the lower edge of the friction pulley. The effective length, *h L*, of the brake *I I* was 15.90 feet, and that of the arms *L'N* and *Ni* of the bell-crank *M*, respectively, 6.015 feet and 12.013 feet, making the effective distance of the point of application of the weight in the scale-pan *l* from the centre of the water-wheel shaft,

$$\frac{15.900 \times 12.013}{6.015} = 31.755 \text{ feet};$$

the circumference of a circle of this radius being 199.523 feet. The speed of the wheel was obtained by means of a clock, driven by a worm at the top of the wheel shaft (not represented on the plate), which struck a bell at intervals of 25 revolutions, the time of each stroke being noted and recorded by an observer with a marine chronometer beating half-seconds.

In making the experiments, a separate observer was appointed to note each item of the required data; each was furnished with a time-keeper adjusted to the chronometer with which the speed of the wheel was noted. The time of making the observation and the reading at the several gauges were usually made and recorded continuously every minute, without reference to whether they would be ultimately used or not. Subsequently, by a comparison of the several note books, the time of the beginning and ending of an experiment was selected so as to insure the greatest uniformity in the readings, indicating that there was, as nearly as practicable, a state of permanence in all the data. A mean of all the observations made at each station, within the time selected, was taken for the data at that station.

The data obtained by observation, and the results of the computations, are shown on the accompanying sheet, headed "Table of Experiments on the Humphrey Turbine at the Tremont and Suffolk Mills."

Most of the columns are sufficiently explained by their respective headings.

Column 4.—Opening of the regulating-gate. The distance given is the chord of a circle of 4.35 feet radius, which is the radius of the outer periphery of the stationary cylindrical casing *BB*. The height of the orifice is 2.086 feet, and is the same for all openings, being the height of the orifices in the stationary cylindrical casing *BB*. The mean width, *w w*, at the smallest section is 0.521 foot, and the average height of the same is 2.130 feet, making the sum of the areas of the smallest sections of the twelve orifices through which the water flows before entering the wheel, 13.317 square feet. The co-efficient of discharge through these smallest sections, when the regulating-gate is fully open, varies with the velocity; when the velocity of the periphery of the wheel is 0.70 of the velocity due the fall acting upon the wheel, the co-efficient is 0.559; when the velocity is 0.73, the co-efficient is 0.551; when 0.75, the co-efficient is 0.546; and when 1.17, the co-efficient is reduced to 0.362. In this last case the wheel was running without load, the brake being loose. When the brake was tightened up so as to prevent the wheel from revolving, the co-efficient of discharge was 0.620.

Column 8.—The weights used were corrected and sealed by the official sealer of weights and measures for the city of Lowell.

Column 9 is found in the following manner: The number in column 7 being divided by the corresponding number in column 6, the quotient is the number of revolutions of the wheel per second. This quotient, multiplied by 199.523, gives the velocity that the point of application of the weight tends to take, or, in other words, the velocity with which the weight would be lifted, if, for an infinitely short time, the brake and the apparatus connecting it with the weight were rigidly connected with the friction pulley; this velocity, multiplied by the weight in the scale-pan (given in column 8), gives the useful effect given in column 9.

Columns 10 and 11 are obtained in the manner explained in the above description of the wheel and apparatus.

Column 12, weight of a cubic foot of water, is taken from Table I in Lowell Hydraulic Experiments.

Column 13.—Total power of the water. This is the product of the corresponding numbers in columns 10 and 12, and the quantity of water discharged by the wheel given in column 11.

Column 14 is the quotient of the corresponding numbers in column

9 divided by those in column 13, and is the ratio of the useful effect of the wheel to the total power of the water expended.

Column 15 gives the velocity of the exterior periphery of the wheel, relatively, to the theoretical velocity of the water due to the fall acting on the wheel.

Column 16.—The quantity of water which would have been discharged by the wheel, if the fall acting on the wheel had been 13 feet. This is deduced from the actual discharge given in column 11 under the corresponding fall given in column 10, corrected for a fall of 13 feet, on the principle that the discharge varies as the square root of the head. These quantities are given to show the variations in the discharge of the wheel depending on its velocity, the effect of the variations in the fall acting upon the wheel being eliminated.

The ratios of the useful effect to the power expended, and the corresponding velocities of the wheel for the several openings of the regulating-gate, are represented by the diagrams on Plate XLIII. The abscissas being the "ratios of the velocity of the exterior circumference of the wheel to the velocity due to the fall acting on the wheel," given in column 15 of the table, and the ordinates, the corresponding "ratios of the useful effect to the power expended," given in column 14. By reference to the table and diagrams, it will be seen that when the speed gate is fully open and the ratio of the velocity of the exterior circumference of the wheel to the velocity due to the fall acting on the wheel is between 70 and 75 per cent., the co-efficient of useful effect is about 82 per cent. As the opening of the gate is reduced, the co-efficient of useful effect diminishes, and also the velocity of the wheel giving the maximum co-efficient of useful effect; thus, when the regulating-gate is one-half open, nearly, the co-efficient of useful effect is about 65 per cent., and the velocity giving the maximum co-efficient of useful effect between 57 and 66 per cent.

1	2	3		4	5						6
		Temperature			Time.						
No. of the experiment.	Date.	of air in wheel-room.	of water in wheel-pit.	Opening of the regulating-gate.	Beginning of the experiment.			Ending of the experiment.			Duration of the experiment.
	1883.	Deg. Fah.	Deg. Fah.	Inches.	Hours.	Min.	Sec.	Hours.	Min.	Sec.	Seconds.
1	Oct. 30, P. M.	67.5	49.0	12.74	5	10	52.5	5	22	4.0	671.5
2	"	"	"	"	5	24	40.0	5	33	3.0	503.0
3	"	"	"	"	5	34	38.0	5	44	41.5	603.5
4	Nov. 1, A. M.	61.5	48.0	"	9	36	51.0	9	55	18.0	1107.0
5	"	"	"	"	9	56	52.0	10	8	19.5	687.5
6	"	"	"	"	10	9	53.0	10	21	45.0	712.0
7	"	"	"	"	10	23	47.5	10	35	3.0	675.5
8	"	"	"	"	10	37	35.5	10	50	17.0	761.5
9	"	"	"	"	10	53	59.0	11	2	29.0	510.0
10	"	"	"	"	11	4	37.0	11	16	24.0	707.0
11	"	"	"	"	11	17	28.5	11	28	49.0	680.5
12	"	"	"	"	11	31	32.5	11	44	5.0	752.5
13	Nov. 1, P. M.	63.0	47.5	"	2	25	48.5	2	37	41.0	712.5
14	"	"	"	"	2	39	52.5	2	51	22.5	690.0
15	"	"	"	"	2	55	48 5	3	4	41.5	583.0
16	"	"	"	"	3	7	59.5	3	19	53.5	714.0
17	"	"	"	"	3	21	37.5	3	27	27.5	350.0
18	"	"	"	"	3	32	52.0	3	39	38.5	406.5
19	"	"	"	"	3	55	48.0	4	5	39.0	591.0
20	"	"	"	10.51	4	16	57.5	4	23	44.5	407.0
21	"	"	"	"	4	26	37.0	4	38	6.5	689.5
22	"	"	"	"	4	40	33.0	4	49	52.0	559.0
23	"	"	"	"	4	51	51.0	5	1	47.0	596.0
24	"	"	"	"	5	2	47.0	5	11	22.5	515.5
25	"	"	"	"	5	13	55.5	5	24	11.5	616.0
26	"	"	"	"	5	25	45.0	5	35	42.0	597.0
27	Nov. 2, A. M.	63.5	46.5	"	8	27	34.5	8	44	44.5	1030.0
28	"	"	"	"	8	46	51.0	8	59	6.0	735.0
29	"	"	"	"	9	0	42.5	9	12	39.0	716.5
30	"	"	"	"	9	15	58.0	9	25	24.5	566.5
31	"	"	"	"	9	28	48.5	9	37	20.5	512.0
32	"	"	"	"	9	39	39.5	9	47	16.0	456.5
33	"	"	"	"	9	50	50.5	10	1	8.0	617.5
34	"	"	"	"	10	4	54.0	10	11	5.0	371.0
35	"	"	"	9.84	10	32	49.5	10	43	14.5	625.0
36	"	"	"	"	10	45	53.5	10	55	30.0	576.5
37	"	"	"	"	10	57	41.0	11	11	23.0	822.0
38	"	"	"	"	11	14	43.0	11	23	10.0	507.0
39	"	"	"	"	11	24	53.0	11	35	17.5	624.5
40	"	"	"	"	11	38	51.5	11	47	14.5	503.0
41	Nov. 2, P. M.	66.5	"	"	2	22	33.5	2	29	10.5	397.0
42	"	"	"	"	2	39	57.5	2	44	19.0	261.5
43	"	"	"	8.69	2	59	36.5	3	14	20.0	883.5
44	"	"	"	"	3	16	29.0	3	29	16.0	767.0
45	"	"	"	"	3	31	28.0	3	41	1.5	573.5
46	"	"	"	"	3	43	54.0	3	52	34.5	520.5
47	"	"	"	"	3	55	32.0	4	3	17.0	465.0
48	"	"	"	"	4	4	30.5	4	12	34.0	483.5
49	"	"	"	6.74	4	34	39.0	4	45	1.0	622.0
50	"	"	"	"	4	47	30.0	5	0	5.5	755.5
51	"	"	"	"	5	1	37.0	5	15	27.5	830.5
52	"	"	"	"	5	16	30.0	5	25	29.0	539.0
53	"	"	"	"	5	26	33.0	5	35	16.0	523.0
54	"	"	"	"	5	37	27.5	5	45	20.5	473.0
55	Nov. 5, A. M.	58.0	44.0	"	9	9	59.5	9	24	32.5	873.0
56	"	"	"	"	9	26	50.0	9	37	11.5	621.5
57	"	"	"	"	9	38	57.0	9	50	10.5	673.5
58	"	"	"	"	9	52	36.5	10	2	20.0	583.5
59	"	"	"	"	10	4	48.5	10	14	11.5	563 0
60	"	"	"	"	10	16	46.0	10	27	8.0	622.0
61	"	"	"	"	10	29	46.5	10	38	34.5	528.0
62	"	"	"	"	10	40	40.5	10	49	10.5	510.0

E OF EXPERIMENTS ON THE HUMPHREY TURBINE AT THE TREMONT AND SUFFOLK MILLS.

	6	7	8	9	10	11					12	13	14	15	16
						Quantity of water discharged.									
						Easterly Weir. Length, 11.22 ft.		Westerly Weir. Length, 10.98 ft.							
	Duration of the experiment.	Total number of revolutions of the wheel during the experiment.	Weight in the scale-pan.	Useful effect, or the friction of the brake.	Total fall acting upon the wheel.	Depth of water on the weir.	Quantity of water passing the weir.	Depth of water on the weir.	Quantity of water passing the weir.	Quantity of water discharged by the wheel.	Weight of a cubic foot of water.	Total power of the water.	Ratio of the useful effect to the power expended.	Ratio of the velocity of the exterior circumference of the wheel to the velocity due the fall acting on the wheel.	Quantity of water which passed the wheel, reduced to a uniform fall of 13 ft.
Sec.	Seconds.		Pounds Avoir.	Pounds raised one foot per second.	Feet.	Feet.	Cubic feet per second.	Feet.	Cubic feet per second.	Cubic feet per second.	Pounds Avoir.	Pounds raised one foot per second.			Cubic feet per second.
4.0	671.5	550	800	130 737	12.542	1.969	104.91	1.948	101.05	205.94	62.371	161 098	0.8115	0.7546	209.67
3.0	503.0	400	830	131 693	12.500	1.983	106.09	1.955	101.59	207.66	"	161 900	0.8134	0.7339	211.77
41.5	603.5	475	840	131 913	12.473	1.985	106.25	1.957	101.74	207.97	"	161 791	0.8153	0.7272	212.32
18.0	1107.0	875	840	1-2 475	12.478	1.987	106.41	1.953	101.43	207.82	62.373	161 744	0.8190	0.7301	212.12
19.5	687.5	550	830	132 483	12.504	1.984	106.17	1.951	101 28	207.43	"	161 777	0.8189	0.7382	211.50
45.0	712.0	575	820	132 128	12.519	1.980	105.86	1.948	101.05	206.89	"	161 550	0.8179	0.7448	210.83
3.0	675.5	550	810	131 588	12.532	1.976	105.47	1.945	100.82	206.27	"	161 233	0.8161	0.7505	210.09
17.0	761.5	625	800	131 007	12.533	1.972	105.15	1.944	100.74	205.87	"	160 933	0.8140	0.7565	209.67
29.0	510.0	400	850	133 015	12.495	1.991	106.73	1.958	101.82	208.53	"	162 518	0.8185	0.7240	212.70
24.0	707.0	550	860	133 486	12.489	1.996	107.13	1.964	102.28	209.39	"	163 110	0.8184	0.7183	213.63
49.0	680.5	525	870	133 919	12.479	2.002	107.61	1.968	102.59	210.18	"	163 594	0.8186	0.7126	214.52
5.0	752.5	575	880	134 164	12.471	2.008	108.16	1.968	102.59	210.73	"	163 917	0.8185	0.7060	215.15
41.0	712.5	550	880	135 536	12.580	2.011	108.40	1.970	102.75	211.13	62.374	165 666	0.8181	0.7101	214.63
22.5	690.0	525	895	135 871	12.568	2.017	108.88	1.972	102.90	211.76	"	166 002	0.8185	0.7003	215.37
41.5	533.0	400	910	136 260	12.574	2.025	109.52	1.980	103.59	213.09	"	167 125	0.8153	0.6906	216 67
53.5	714.0	525	930	136 438	12.564	2.034	110.24	1.980	103.59	213.81	"	167 556	0.8143	0.6769	217.49
27.5	350.0	250	955	136 103	12.540	2.051	111.68	1.985	103.98	215.64	"	168 667	0.8069	0.6582	219.56
38.5	406.5	275	1000	134 979	12.499	2.071	113.37	1.984	103.90	217.25	"	169 371	0.7969	0.6244	221.56
39.0	591.0	775	0	0	13.341	1.535	71.84	1.520	69.28	141.10	"	117 414	.....	1.1715	139.29
44.5	407.0	350	650	111 527	12.839	1.802	91.71	1.804	89.90	181.59	"	145 421	0.7669	0.7831	182.72
6.5	689.5	600	635	110 251	12.829	1.794	91.04	1.795	89.17	180.19	"	144 187	0.7646	0.7927	181.39
52.0	559.0	475	660	111 897	12.785	1.808	92.17	1.808	90.20	182.35	"	145 415	0.7695	0.7754	183.88
47.0	596.0	500	675	112 985	12.779	1.818	92.93	1.817	90.86	183.77	"	146 479	0.7713	0.7657	185.35
22.5	515.5	425	695	114 324	12.777	1.829	93.76	1.825	91.46	185.20	"	147 596	0.7746	0.7526	186.81
11.5	616.0	500	715	115 795	12.762	1.839	94.60	1.832	91 98	186.56	"	148 505	0.7797	0.7414	188.29
42.0	597.0	475	735	116 681	12.761	1.847	95.21	1.836	92.28	187.47	"	149 218	0.7820	0.7268	189.22
44.5	1030.0	825	735	117 462	12.803	1.845	95.05	1 838	92.50	187.53	62.375	149 759	0.7843	0.7304	188.97
6.0	735.0	575	755	117 848	12.755	1.852	95.59	1.838	92.50	188.07	"	149 627	0.7876	0.7147	189.87
39.0	716.5	550	775	118 697	12.727	1.860	96.20	1.846	93.10	189.28	"	150 259	0.7900	0.7021	191.30
24.5	566.5	425	795	119 001	12.713	1.869	96.97	1.851	93.47	190.42	"	150 998	0.7881	0.6866	192.56
20.5	512.0	375	820	119 831	12.701	1.880	97.82	1.864	94.52	192 32	"	152 361	0.7865	0.6706	194.57
16.0	456.5	325	845	120 031	12.678	1.893	98.82	1.867	94.74	193.54	"	153 050	0.7843	0.6524	195.98
8.0	617.5	425	870	119 471	12.631	1.910	100.21	1.869	94.89	195.08	"	153 695	0.7773	0.6319	197.91
5.0	371.0	250	900	121 004	12.733	1.926	101.46	1.877	95.50	196.94		156 414	0.7736	0.6162	198.99
14.5	625.0	500	700	111 733	12.983	1.795	91.12	1.795	89.17	180.27	"	145 985	0.7654	0.7245	180.39
30.0	576.5	450	725	112 913	12.959	1.804	91.87	1.801	89.68	181.53	"	146 754	0.7695	0.7075	181.82
23.0	822.0	625	750	113 779	12.942	1.814	92.62	1.807	90.12	182.72	"	147 502	0.7714	0.6896	183.13
19.0	507.0	375	775	114 372	12.915	1.825	93.46	1.813	90.57	184.01	"	148 234	0.7716	0.6716	184.61
17.5	624.5	450	800	115 017	12.884	1.836	94.30	1.823	91.31	185.59	"	149 147	0.7712	0.6550	186.42
14.5	503.0	350	825	114 537	12.852	1.852	95.59	1.828	91.68	187.25	"	150 108	0.7630	0.6333	188.33
10.5	397.0	275	825	114 022	12.852	1.856	95.90	1.825	91.46	187.34	"	150 180	0.7592	0.6305	188.42
19.0	261.5	175	850	113 495	12.805	1.870	97.04	1.828	91.68	188.70	"	150 717	0.7530	0.6102	190.13
20.0	883.5	675	650	99 084	13.103	1.703	84.13	1.708	82.69	166.80	"	136 326	0.7268	0.6887	166.14
16.0	767.0	600	625	97 550	13.107	1.690	83.18	1.699	82.04	165.20	"	135 059	0.7223	0.7050	164.52
1.5	573.5	425	675	99 805	13.075	1.712	84.79	1.714	83.13	167.90	"	136 931	0.7289	0.6687	167.42
34.5	520.5	375	700	100 624	13.070	1.722	85.60	1.720	83.56	169.14	"	137 890	0.7297	0.6503	168.69
17.0	465.0	325	725	101 102	13.035	1.732	86.34	1.727	84.13	170.45	"	138 586	0.7295	0.6184	170.22
34.0	483.5	325	750	100 587	12.975	1.748	87.53	1.731	84.42	171.93	"	139 146	0.7229	0.6089	172.10
1.0	622.0	525	400	67 363	13.456	1.455	66.28	1.461	65.26	131.52	"	110 387	0.6102	0.7508	129.27
5.5	755.5	625	415	68 499	13.418	1.460	66.62	1.466	65.59	132.19	"	110 636	0.6191	0.7369	130.11
27.5	830.5	675	430	69 731	13.447	1.467	67.09	1.474	66.12	133.19	"	111 714	0.6242	0.7232	130.96
29.0	539.0	425	455	71 582	13.449	1.478	67.84	1.484	66.79	134.61	"	112 922	0.6339	0.7016	132.34
16.0	523.0	400	480	73 247	13.426	1.488	68.53	1.496	67.60	136.11	"	113 985	0.6426	0.6811	133.93
20.5	473.0	350	505	74 557	13.411	1.501	69.48	1.508	68.47	137.93	"	115 380	0.6462	0.6593	135.80
32.5	373.0	650	505	75 021	13.489	1.496	69.08	1.507	68.40	137.46	62.379	115 663	0.6486	0.6615	134.95
11.5	621.5	450	525	75 844	13.445	1.504	69.69	1.516	69.01	138.68	"	116 309	0.6521	0.6443	136.37
10.5	673.5	475	545	76 691	13.440	1.512	70.24	1.524	69.55	139.77	"	117 180	0.6545	0.6277	137.46
20.0	583.5	400	565	77 279	13.436	1.519	70.73	1.532	70.10	140.81	"	118 016	0.6548	0.6102	138.51
11.5	563 0	375	585	77 745	13.427	1.527	71.28	1.540	70.71	141.97	"	118 909	0.6538	0 5931	139.69
8.0	622.0	400	605	77 628	13.408	1.535	71.84	1.545	71.05	142.87	"	119 493	0.6496	0.5731	140.68
34.5	528.0	325	625	76 758	13.381	1.541	72.32	1.545	71.05	143.35	"	119 653	0.6415	0.5491	141.29
10.5	510.0	300	645	75 701	13.355	1.546	72.67	1.546	71.12	143.77	"	119 771	0.6321	0.5252	141.85

11				12	13	14	15	16	17
Quantity of water discharged.				Weight of a cubic foot of water.	Total power of the water.	Ratio of the useful effect to the power expended.	Ratio of the velocity of the exterior circumference of the wheel to the velocity due the fall acting on the wheel.	Quantity of water which passed the wheel, reduced to a uniform fall of 13 ft.	Remarks.
.	Westerly Weir. Length, 10.98 ft.		Quantity of water discharged by the wheel.						
tity of passing weir.	Depth of water on the weir.	Quantity of water passing the weir.							
ic feet econd.	Feet.	Cubic feet per second.	Cubic feet per second.	Pounds Avoir.	Pounds raised one foot per second.			Cubic feet per second.	
4.91	1.948	101.05	205.94	62.371	161 098	0.8115	0.7546	209.67	
5.09	1.955	101.59	207.66	"	161 900	0.8134	0.7339	211.77	
5.25	1.957	101.74	207.97	"	161 791	0.8153	0.7272	212.32	
5.41	1.953	101.43	207.82	62.373	161 744	0.8190	0.7301	212.12	
5.17	1.951	101 28	207.43	"	161 777	0.8189	0.7382	211.50	
5.86	1.948	101.05	206.89	"	161 550	0.8179	0.7448	210.83	
5.47	1.945	100.82	206.27	"	161 233	0.8161	0.7505	210.09	
5.15	1.944	100.74	205.87	"	160 933	0.8140	0.7565	209.67	
6.73	1.958	101.82	208.53	"	162 518	0.8185	0.7240	212.70	
7.13	1.964	102.28	209.39	"	163 110	0.8184	0.7183	213.63	
7.61	1.968	102.59	210.18	"	163 594	0.8186	0.7126	214.52	Speed Gate fully opened.
8.16	1.968	102.59	210.73	"	163 917	0.8185	0.7060	215.15	
8.40	1.970	102.75	211.13	62.374	165 606	0.8181	0.7101	214.63	
8.88	1.972	102.90	211.76	"	166 002	0.8185	0.7003	215.37	
9.52	1.980	103.59	213.09	"	167 125	0.8153	0.6906	216 67	
0.24	1.980	103.59	213.81	"	167 556	0.8143	0.6769	217.49	
1.68	1.985	103.98	215.64	"	168 667	0.8069	0.6582	219.56	
3.37	1.984	103.90	217.25	"	169 371	0.7969	0.6244	221.56	
1.84	1.520	69.28	141.10	"	117 414	.....	1.1715	139.29	Brake loose on friction pulley.
1.71	1.804	89.90	181.59	"	145 421	0.7669	0.7831	182.72	
1.04	1.795	89.17	180.19	"	144 187	0.7646	0.7927	181.39	
2.17	1.808	90.20	182.35	"	145 415	0.7695	0.7754	183.88	
2.93	1.817	90.86	183.77	"	146 479	0.7713	0.7657	185.35	
3.76	1.825	91.46	185.20	"	147 596	0.7746	0.7526	186.81	
4.60	1.832	91.98	186.56	"	148 505	0.7797	0.7414	188.29	
5.21	1.836	92.28	187.47	"	149 218	0.7820	0.7268	189.22	
5.05	1 838	92.50	187.53	62.375	149 759	0.7843	0.7304	188.97	Opening of Speed Gate 82.50 per cent. of full gate.
5.59	1.838	92.50	188.07	"	149 627	0.7876	0.7147	189.87	
6.20	1.846	93.10	189.28	"	150 259	0.7900	0.7021	191.30	
6.97	1.851	93.47	190.42	"	150 998	0.7881	0.6866	192.56	
7.82	1.864	94.52	192 32	"	152 361	0.7865	0.6706	194.57	
8.82	1.867	94.74	193.54	"	153 050	0.7843	0.6524	195.98	
0.21	1.869	94.89	195.08	"	153 695	0.7773	0.6319	197.91	
1.46	1.877	95.50	196.94	"	156 414	0.7736	0.6162	198.99	
1.12	1.795	89.17	180.27	"	145 985	0.7654	0.7245	180.39	
1.87	1.801	89.68	181.53	"	146 734	0.7695	0.7075	181.82	
2.62	1.807	90.12	182.72	"	147 502	0.7714	0.6896	183.13	
3.46	1.813	90.57	184.01	"	148 234	0.7716	0.6716	184.61	Opening of Speed Gate 77.24 per cent. of full gate.
4.30	1.823	91.31	185.59	"	149 147	0.7712	0.6550	186.42	
5.59	1.828	91.68	187.25	"	150 108	0.7630	0.6333	188.33	
5.90	1.825	91.46	187.34	"	150 180	0.7592	0.6305	188.42	
7.04	1.828	91.68	188.70	"	150 717	0.7530	0.6102	190.13	
4.13	1.708	82.69	166.80	"	136 326	0.7268	0.6887	166.14	
3.18	1.699	82.04	165.20	"	135 059	0.7223	0.7050	164.52	
4.79	1.714	83.13	167.90	"	136 941	0.7289	0.6687	167.42	Opening of Speed Gate 68.21 per cent. of full gate.
5.60	1.720	83.56	169.14	"	137 890	0.7297	0.6503	168.69	
6.34	1.727	84.13	170.45	"	138 586	0.7295	0.6184	170.22	
7.53	1.731	84.42	171.93	"	139 146	0.7229	0.6089	172.10	
6.28	1.461	65.26	131.52	"	110 387	0.6102	0.7508	129.27	
6.62	1.466	65.59	132.19	"	110 636	0.6191	0.7369	130.11	
7.09	1.474	66.12	133.19	"	111 714	0.6242	0.7232	130.96	
7.84	1.484	66.79	134.61	"	112 922	0.6339	0.7016	132.34	
8.53	1.496	67.60	136.11	"	113 985	0.6426	0.6811	133.93	
9.48	1.508	68.47	137.93	"	115 380	0.6462	0.6593	135.80	
9.08	1.507	68.40	137.46	62.379	115 663	0.6486	0.6615	134.95	Opening of Speed Gate 52.90 per cent. of full gate.
9.69	1.516	69.01	138.68	"	116 309	0.6521	0.6443	136.37	
0.24	1.524	69.55	139.77	"	117 180	0.6545	0.6277	137.46	
0.73	1.532	70.10	140.81	"	118 016	0.6548	0.6102	138.51	
1.28	1.540	70.71	141.97	"	118 909	0.6538	0 5931	139.69	
1.84	1.545	71.05	142.87	"	119 493	0.6496	0.5731	140.68	
2.32	1.545	71.05	143.35	"	119 653	0.6415	0.5491	141.29	
2.67	1.546	71.12	143.77	"	119 771	0.6321	0.5252	141.85	

1	2	3		4	5						6
		Temperature			Time.						
No. of the experiment.	Date.	of air in wheel-room.	of water in wheel-pit.	Opening of the regulating gate.	Beginning of the experiment.			Ending of the experiment.			Duration of the experiment.
	1883.	Deg. Fah.	Deg. Fah.	Inches.	Hours.	Min.	Sec.	Hours.	Min.	Sec.	Seconds.
63	Nov. 5, A. M.	58.0	44.0	5.18	11	9	54.5	11	18	8.0	493.5
64	"	"	"	"	11	23	39.0	11	32	5.0	506.0
65	"	"	"	"	11	34	38.0	11	42	13.5	455.5
66	Nov. 7, A. M.	68.0	45.0	"	8	24	56.5	8	36	0.5	664.0
67	"	"	"	"	8	38	57.5	8	48	10.0	552.5
68	"	"	"	"	8	51	31.0	8	59	30.5	479.5
69	"	"	"	"	9	1	54.0	9	11	29.5	575.5
70	"	"	"	"	9	12	44.5	9	21	32.0	527.5
71	"	"	"	"	9	22	49.0	9	33	23.0	634.0
72	"	"	"	"	9	34	45.0	9	42	24.5	459.5
73	"	"	"	"	9	43	51.0	9	51	12.5	441.5
74	"	"	"	3.56	10	12	51.0	10	20	2.0	431.0
75	"	"	"	"	10	21	52.0	10	31	17.0	565.0
76	"	"	"	"	10	33	45.5	10	42	15.5	510.0
77	"	"	"	"	10	43	47.0	10	54	8.0	621.0
78	"	"	"	"	10	56	47.5	11	5	27.0	519.5
79	"	"	"	"	11	7	41.5	11	18	29.5	648.0
80	"	"	"	"	11	20	49.5	11	26	12.5	323.0
81	"	"	"	"	11	33	33.5	11	44	17.0	643.5
82	Nov. 7, P. M.	71.0	"	"	2	39	57.0	2	47	31.0	454.0
83	"	"	"	"	2	48	49.0	2	54	5.5	316.5
84	"	"	"	"	3	11	49.0	3	16	0.5	251.5
85	"	"	"	"	3	18	54.0	3	24	4.5	310.5
86	"	"	"	"	3	26	20.0	3	33	13.5	413.5
87	"	"	"	2.44	3	53	45.5	3	56	5.0	139.5
88	"	"	"	"	3	58	59.0	4	6	1.5	422.5
89	"	"	"	"	4	7	33.0	4	16	31.0	538.0
90	"	"	"	"	4	18	42.0	4	26	26.5	464.5
91	"	"	"	"	4	27	34.0	4	36	15.0	521.0
92	"	"	"	"	4	37	25.5	4	46	30.5	545.0
93	"	"	"	"	4	48	23.0	4	56	37.0	494.0
94	"	"	"	"	4	57	54.5	5	6	29.5	515.0
95	"	"	"	"	5	7	51.0	5	16	8.5	497.5
96	"	"	"	"	5	18	59.5	5	26	15.0	435.5
97	"	"	"	"	5	28	29.5	5	35	21.5	412.0
98	"	"	"	"	5	36	55.5	5	44	13.0	437.5
99	Nov. 8 A. M.	58.5	44.0	9.84	8	33	36.0	8	44	7.0	631.0
100	"	"	"	"	8	46	43.5	8	56	29.5	586.0
101	"	"	"	"	8	59	32.5	9	9	6.0	573.5
102	"	"	"	"	9	10	36.0	9	20	27.0	591.0
103	"	"	"	"	9	22	52.0	9	33	21.5	629.5
104	"	"	"	8.69	9	56	46.5	10	7	29.5	643.0
105	"	"	"	"	10	9	36.5	10	16	25.0	408.5
106	"	"	"	"	10	21	39.0	10	31	26.5	587.5
107	"	"	"	"	10	33	58.5	10	40	0.5	362.0
108	"	"	"	"	10	43	59.0	10	51	21.0	442.0
109	"	"	"	"	10	53	46.0	11	3	15.0	569.0
110	"	"	"	1.45	11	22	38.0	11	28	18.5	340.5
111	"	"	"	"	11	30	54.5	11	36	17.0	322.5
112	"	"	"	"	11	38	26.0	11	44	26.0	360.0
113	"	"	"	"	11	46	30.0	11	50	30.0	240.0

RIMENTS ON THE HUMPHREY TURBINE AT THE TREMONT AND SUFFOLK MILLS—(*Continued*).

6	7	8	9	10	11					12	13	14	15	16	17
					Quantity of water discharged.										
					Easterly Weir. Length, 11.22 ft.		Westerly Weir. Length, 10.98 ft.								
Duration of the experiment.	Total number of revolutions of the wheel during the experiment.	Weight in the scale-pan.	Useful effect, or the friction of the brake.	Total fall acting upon the wheel.	Depth of water on the weir.	Quantity of water passing the weir.	Depth of water on the weir.	Quantity of water passing the weir.	Quantity of water discharged by the wheel.	Weight of a cubic foot of water.	Total power of the water.	Ratio of the useful effect to the power expended.	Ratio of the velocity of the exterior circumference of the wheel to the velocity due the fall acting on the wheel.	Quantity of water which passed the wheel, reduced to a uniform fall of 13 ft.	Remarks.
Seconds.		Pounds Avoir.	Pounds raised one foot per second.	Feet.	Feet.	Cubic feet per second.	Feet.	Cubic feet per second.	Cubic feet per second.	Pounds Avoir.	Pounds raised one foot per second.			Cubic feet per second.	
493.5	375	350	53 065	13.688	1.300	55.86	1.303	54.86	110.70	62.379	94 520	0.5614	0.6702	107.88	
506.0	400	325	51 261	13.671	1.291	55.29	1.294	54.29	109.56	"	93 431	0.5486	0.6976	106 84	
455.5	375	300	49 278	13.667	1.280	54.59	1.286	53.79	108.36	"	92 381	0.5334	0.7266	105.68	
664.0	550	300	49 580	13.774	1.283	54.78	1.292	54.17	108.93	62.378	93 592	0.5297	0.7282	105.83	
552.5	475	275	47 172	13.755	1.272	54.08	1.283	53.61	108.67	"	93 240	0.5059	0.7564	105.65	
479.5	350	375	54 614	13.696	1.313	56.70	1.319	55.87	112.55	"	96 155	0.5680	0.6436	109.65	Opening of Speed Gate 40.66 per cent. of full gate.
575.5	400	400	55 471	13.623	1.320	57.15	1.331	56.63	113.76	"	96 670	0 5738	0.6145	111.13	
527.5	350	425	56 264	13.604	1.329	57.73	1.342	57.38	115.09	"	97 664	0.5761	0.5870	112.51	
634.0	400	450	56 647	13.569	1.336	58.19	1.349	57.83	116.00	"	98 183	0.5770	0.5589	113.54	
459.5	275	475	56 720	13.543	1.343	58.70	1.353	58.09	116.77	"	98 646	0.5750	0.5306	114.41	
441.5	250	500	56 490	13.489	1.351	59.23	1.358	58.41	117.62	"	98 967	0.5708	0.5031	115.47	
431.0	400	100	18 517	14.069	1.032	39.39	1.056	39.89	79.26	"	69 558	0.2662	0.8073	76.19	
565.0	500	130	22 954	14.029	1.042	39.96	1.054	39.78	79.72	"	69 763	0.3290	0.7709	76.74	
510.0	425	160	26 603	13.989	1.053	40.59	1.061	40.23	80.80	"	70 507	0.3773	0.7270	77 89	
621.0	500	175	28 113	13.958	1.058	40.88	1.066	40.51	81.37	"	70 847	0.3968	0.7032	78.53	
519.5	400	195	29 957	13.998	1.067	41.46	1.075	41.02	82.46	"	72 001	0.4161	0.6715	79.47	
648.0	475	215	31 445	13.976	1.073	41.80	1.083	41.48	83.26	"	72 586	0.4332	0.6398	80.30	Opening of Speed Gate 27.94 per cent. of full gate.
323.0	225	235	32 662	13.936	1.080	42.21	1.092	42 60	84.19	"	73 186	0.4463	0.6089	81.31	
643.5	425	255	33 603	13.872	1.086	42.56	1.097	42.29	84.83	"	73 404	0.4578	0.5786	82.12	
454.0	300	255	33 620	13.880	1.083	42.39	1.096	42.23	84.60	"	73 247	0.4590	0.5787	81.87	
316.5	200	275	34 672	13.866	1.092	42.92	1.102	42.57	85.47	"	73 926	0.4690	0.5537	82.76	
251.5	150	295	35 105	13.793	1.098	43.27	1.107	42.86	86.11	"	74 087	0.4738	0.5240	83.60	
310.5	175	320	35 985	13.776	1.109	43.92	1.115	43.32	87.22	"	74 950	0.4801	0.4955	84.73	
413.5	225	335	36 370	13.760	1.115	44.27	1.120	43.62	87.87	"	75 421	0.4822	0.4786	85.41	
139.5	125	0	0	14.043	0.830	28.35	0.871	29.87	58.20	"	... ...	.....	0.7802	56.00	Brake loose on friction pulley.
422.5	350	50	8 264	13.998	0.844	29.07	0.873	29.97	59.02	"	51 534	0.1604	0.7225	56.88	
538.0	425	70	11 033	13.974	0.850	29.38	0.876	30.13	59.49	"	51 856	0 2128	0.6895	57.38	
464.5	350	85	12 779	13.958	0.856	29.74	0.878	30.23	59.95	"	52 197	0.2448	0.6581	57.86	
521.0	375	100	14 361	13.954	0.861	30.00	0.879	30.28	60.26	"	52 452	0.2738	0.6287	58.16	
545.0	375	115	15 788	13.922	0.866	30.26	0.881	30.38	60.62	"	52 644	0.2999	0.6017	58.58	Opening of Speed Gate 19.15 per cent. of full gate.
494.0	325	130	17 064	13.894	0.871	30.52	0.884	30.54	61.04	"	52 902	0.3226	0.5759	59.04	
515.0	325	145	18 257	13.879	0.877	30.84	0.885	30.59	61.41	"	53 165	0.3434	0.5527	59.43	
497.5	300	160	19 250	13.866	0.882	31.10	0.889	30.80	61.88	"	53 522	0.3597	0.5284	59.92	
435.5	250	175	20 044	13.846	0.887	31.36	0.894	31.06	62.40	"	53 894	0.3719	0.5034	60.46	
412.0	225	190	20 703	13.819	0.893	31.68	0.898	31.26	62.92	"	54 237	0.3817	0.4794	61.03	
437.5	225	205	21 035	13.793	0.896	31.84	0.902	31.47	63.29	"	54 454	0.3863	0.4518	61.44	
631.0	500	700	110 670	12.901	1.791	90.82	1.791	88.87	179.67	62.379	144 590	0.7654	0.7198	180.36	
586.0	475	675	109 167	12.880	1.781	90.06	1.786	88.51	178.55	"	143 455	0.7610	0.7370	179.38	Opening of Speed Gate 77.24 per cent. of full gate.
573.5	475	650	107 415	12.882	1.764	88.79	1.775	87.70	176.47	"	141 805	0.7575	0.7530	177.28	
591.0	500	625	105 501	12.866	1.751	87.75	1.762	86.74	174.47	"	140 024	0.7534	0.7696	175.38	
629.5	550	600	104 595	13.022	1.743	87.15	1.750	85.80	172.93	"	140 471	0.7446	0.7900	172.78	
643.0	500	625	96 969	13.008	1.681	82.45	1.696	81.83	164.26	"	133 285	0.7275	0.7035	164.21	
408.5	325	600	95 244	12.984	1.669	81.58	1.683	80.90	162.46	"	131 581	0.7238	0.7204	162.56	
587.5	475	575	92 757	12.912	1.656	80.64	1.669	79.84	160.46	"	129 241	0.7177	0.7342	161.01	Opening of Speed Gate 68.21 per cent. of full gate.
362.0	300	550	90 943	12.887	1.645	79.84	1.657	78.98	158.80	"	127 656	0.7124	0.7533	159.49	
442.0	375	525	88 871	12.870	1.631	78.83	1.642	77.92	156.73	"	125 826	0.7063	0.7717	157.52	
569.0	500	500	87 664	13.011	1.622	78.12	1.632	77.22	155.32	"	126 060	0.6954	0.7949	155.25	
340.5	225	0	0	14.400	0.621	18.33	0.655	19.43	37.74	"	... ...	.....	0.5682	35.86	Brake loose on friction pulley.
322.5	200	25	3 093	14.383	0.624	18.46	0.656	19.47	37.91	"	34 013	0.0909	0.5336	36.04	Opening of Speed Gate 11.38 per cent. of full gate.
360.0	200	50	5 542	14.327	0.632	18.82	0.657	19.51	38.31	"	34 238	0.1619	0.4789	36.49	
240.0	0	....	0	14.275	0.656	19.90	0.660	19.65	39.53	"	... ...	.....	.....	37.72	Wheel prevented from revolving by the brake.

PLATE XLI.
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VOL. XIII NO. CCLXXXVIII
FRANCIS ON
HUMPHREY TURBINE

Humphrey Turbine Water Wheel

at the

Tremont & Suffolk Mills,

Lowell, Mass.

Scale for Figure 1. 4 feet.

Scale for Figure 2. 2 feet.

Fig. 2.

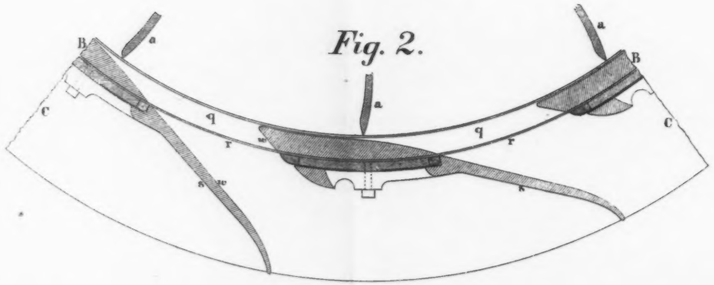
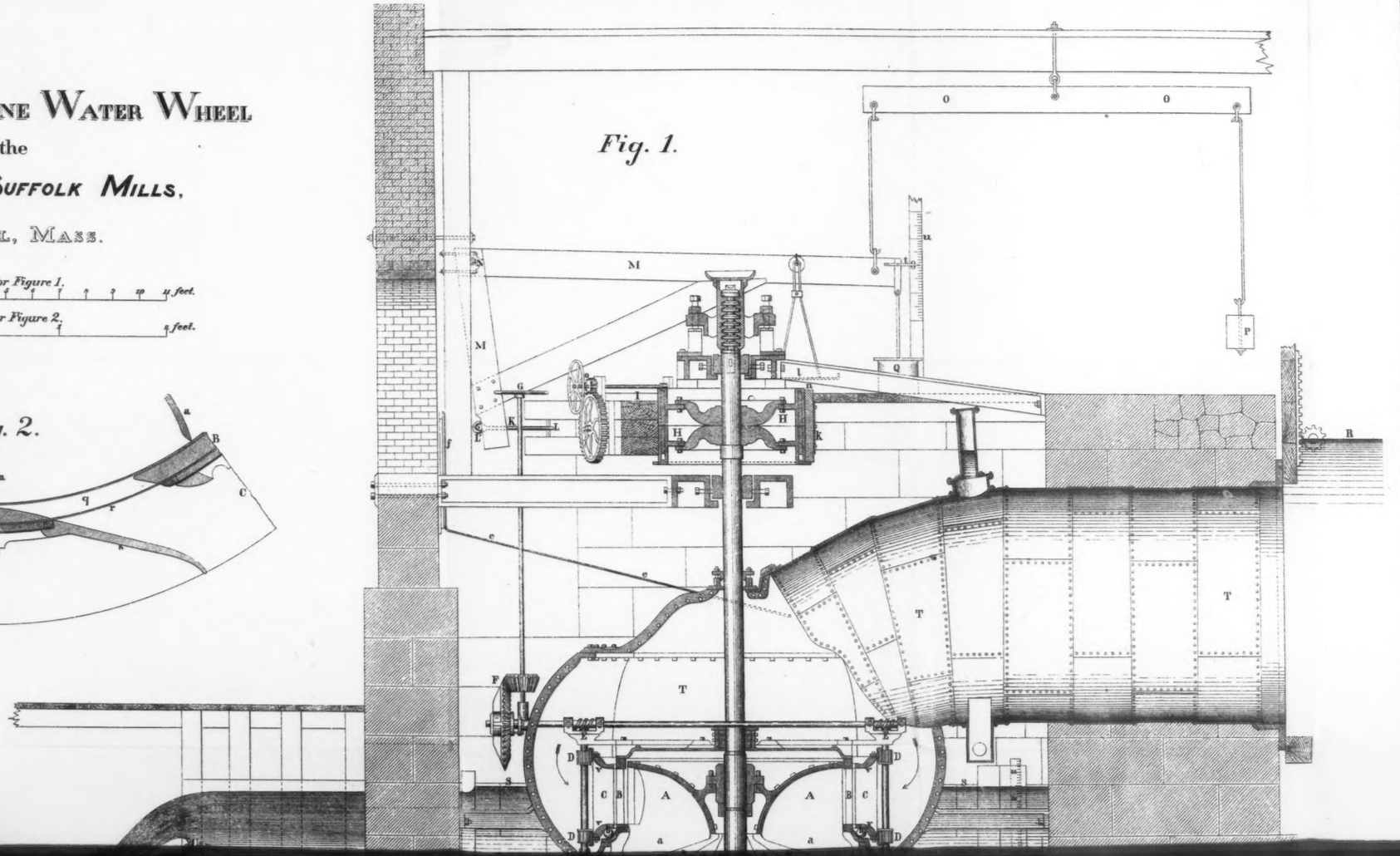


Fig. 1.



LOWELL, MASS.

Scale for Figure 1.

Scale for Figure 2.

Fig. 2.

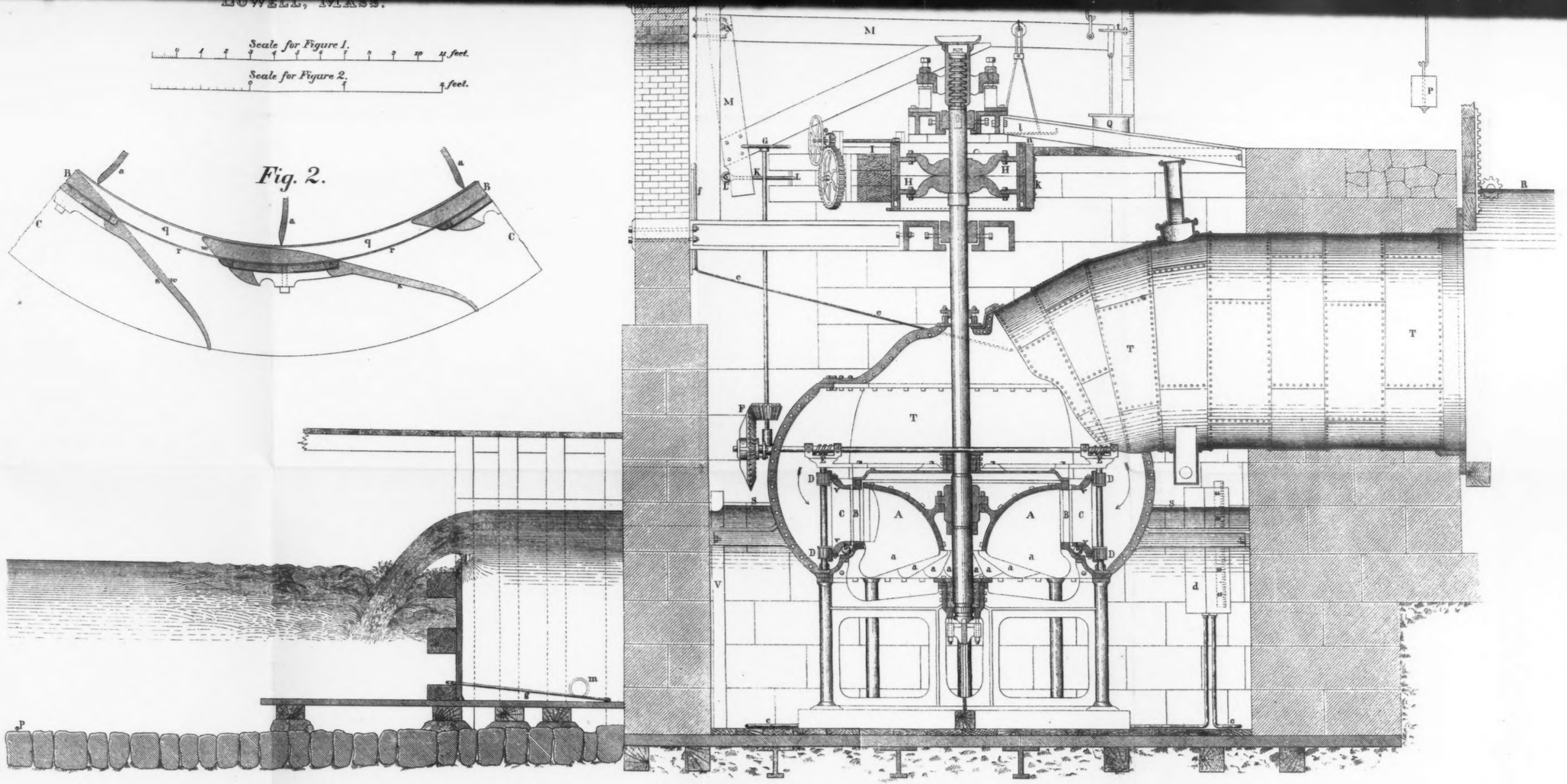


PLATE XLII
TRANS. AM. SOC. CIV. ENG'RS
VOL. XIII NO. CCLXXXVIII
FRANCIS ON
HUMPHREY TURBINE

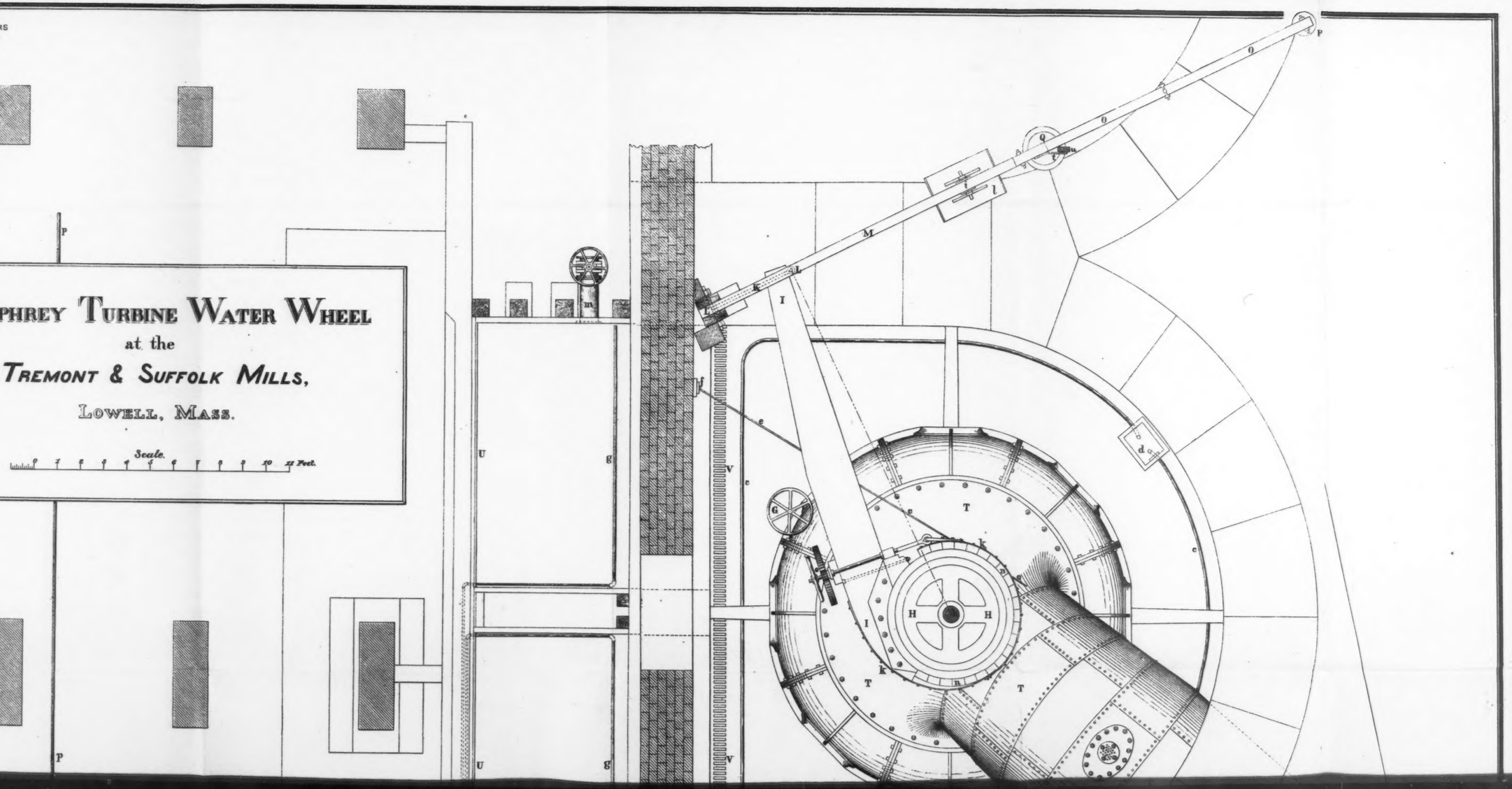
HUMPHREY TURBINE WATER WHEEL

at the

TREMONT & SUFFOLK MILLS,

LOWELL, MASS.

Scale.

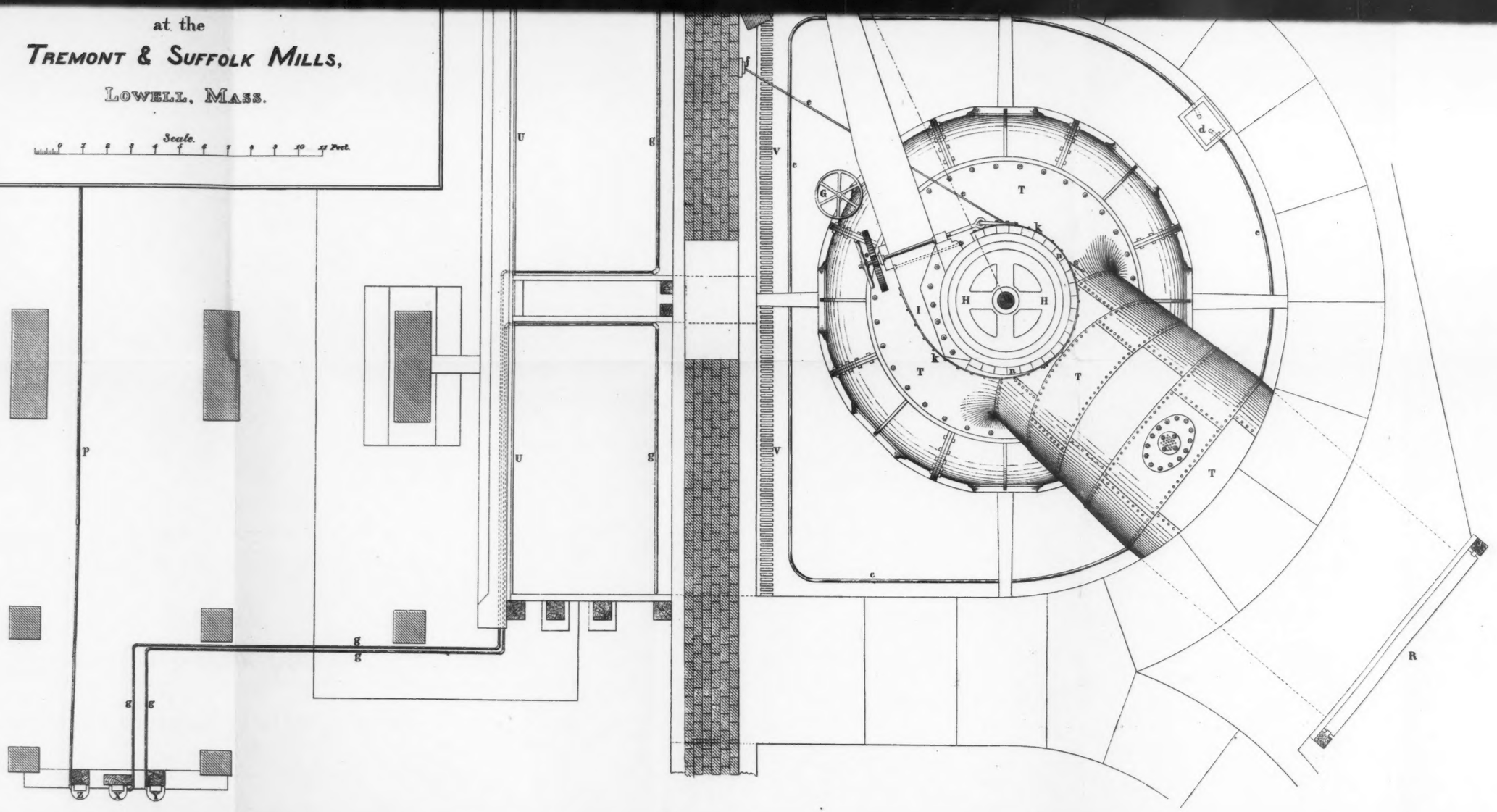


at the

TREMONT & SUFFOLK MILLS,

LOWELL, MASS.

Scale.



W.E.BADGER, DEL.

PLATE XLIII
TRANS. AM. SOC. CIV. ENG'RS
VOL. XIII NO. CCLXXXVIII
FRANCIS ON
HUMPHREY TURBINE

Diagram of the experiments on the Humphrey Turbine at the Tremont and Suffolk Mills.

Ratio of the useful effect to the power expended.

0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80

Ratio of the velocity of the exterior circumference of the wheel to the velocity

0.40 0.50 0.60 0.70

Opening of spindjate 1.45 inches or 11.31 per ct. of full gate.

Opening of spindjate 2.44 inches or 19.35 per ct. of full gate.

Opening of spindjate 3.55 inches or 27.68 per ct. of full gate.

Opening of spindjate 5.48 inches or 42.74 per ct. of full gate.

Opening of spindjate 6.24 inches or 48.66 per ct. of full gate.

Opening of spindjate 8.80 inches or 68.61 per ct. of full gate.

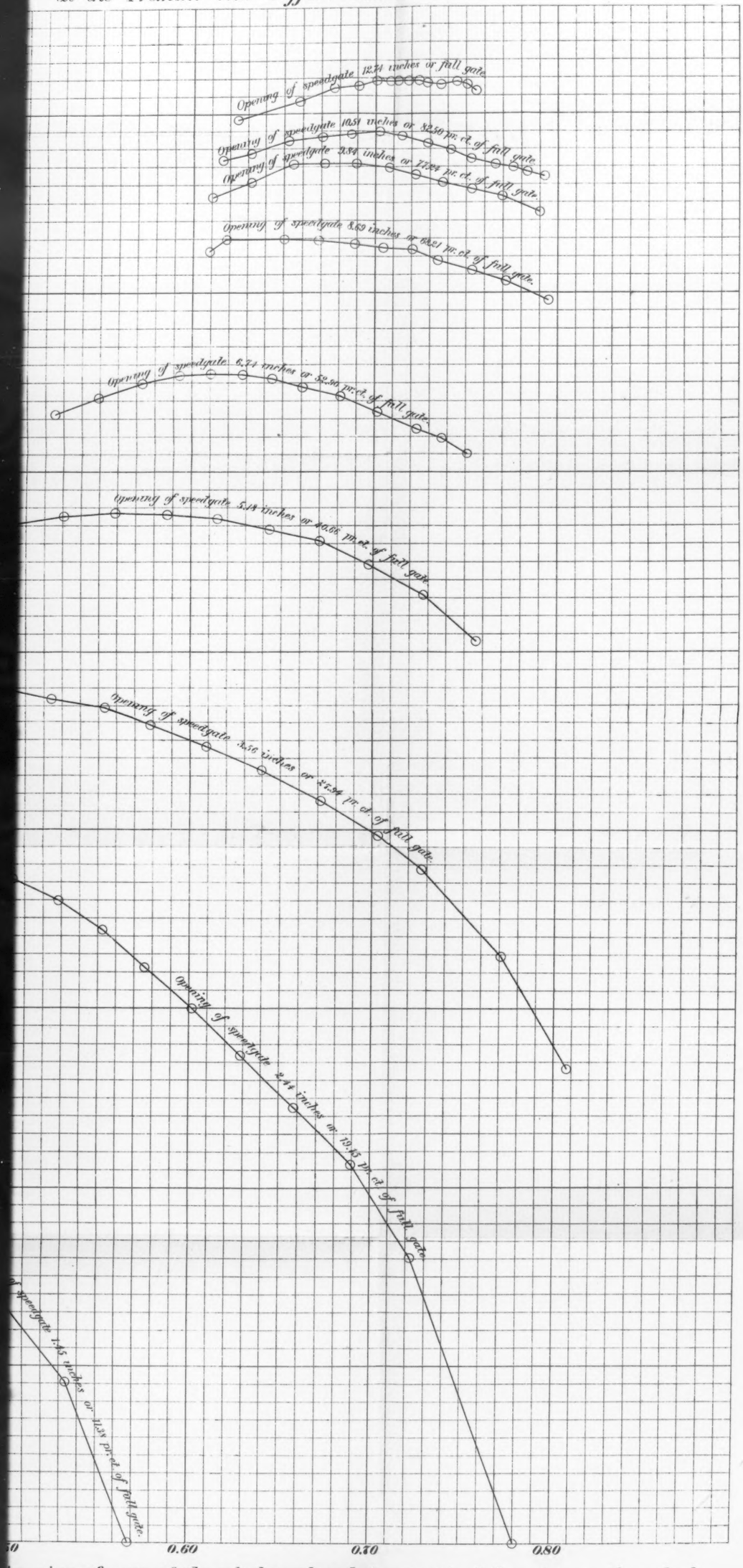
Opening of spindjate 10.24 inches or 79.84 per ct. of full gate.

Opening of spindjate 10.51 inches or 81.93 per ct. of full gate.

Opening of spindjate 10.82 inches or 84.37 per ct. of full gate.

Opening of spindjate 12.82 inches or 100 per ct. of full gate.

the experiments on the Humphrey Turbine Water Wheel at the Tremont and Suffolk Mills.


Opening of speedgate 12.74 inches or full gate.
Opening of speedgate 10.51 inches or 82.50 pr. ct. of full gate.
Opening of speedgate 9.84 inches or 77.24 pr. ct. of full gate.
Opening of speedgate 8.69 inches or 68.21 pr. ct. of full gate.
Opening of speedgate 6.74 inches or 52.90 pr. ct. of full gate.
Opening of speedgate 5.19 inches or 40.66 pr. ct. of full gate.
Opening of speedgate 3.56 inches or 27.94 pr. ct. of full gate.
Opening of speedgate 2.44 inches or 19.15 pr. ct. of full gate.
Opening of speedgate 1.45 inches or 11.38 pr. ct. of full gate.
0.50
0.60
0.70
0.80

or circumference of the wheel to the velocity due the fall acting on the wheel.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

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TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced i any of its publications.

CCLXXXIX.

(Vol. XIII.—September, 1884.)

EXPERIMENTS ON THE FLOW OF WATER OVER SUBMERGED WEIRS.

By JAMES B. FRANCIS, Past President Am. Soc. C. E.

READ AT THE ANNUAL CONVENTION, JUNE 10TH, 1884.

The usual formula for computing the flow over a weir, the crest of which is below the level of the surface of the water on the down-stream side of the weir, is

$$Q = C\tfrac{2}{3} L \sqrt{2g}(H'' - H''')^{\frac{3}{2}} + C' L H''' \sqrt{2g\,(H'' - H''')}$$

in which

Q = the quantity of water discharged in cubic feet per second.

L = the length of the weir.

H'' = the height of the water on the up-stream side of the weir above its crest.

H''' = the height of the water on the down-stream side of the weir above its crest.

g = the velocity acquired by a body at the end of the first second of its fall in a vacuum.

C and C', co-efficients to be determined by experiment.

The first term of the second member of this equation is the usual formula for computing the flow over a weir in which there is no obstruction to the flow from the water on the down-stream side, using for the depth on the weir $H'' - H'''$; that is, the part of the whole depth, H'', which is above the level of the water on the down-stream side, is treated as a regular weir.

The second term of the second member of the equation is the usual formula for the flow through an orifice of the length L and height H''', through which the water is discharged under a head $H'' - H'''$.

The apparatus arranged for testing a turbine water-wheel at the Tremont and Suffolk mills in 1883, and represented in Plates XLI and XLII in Paper CCLXXXVIII of the Transactions of this Society, afforded a favorable opportunity for testing the above formula and determining the values of the co-efficients C and C'; the only additions required to that described in Paper CCLXXXVIII being the hook-gauge Z and the pipe pp for determining the exact height of the water on the down-stream side of the weirs. This pipe was 1.5 inches in diameter, and the part immediately opposite the weirs had 16 holes on its upper side, placed at equal distances, $\frac{3}{16}$ inch in diameter, which were its only openings, except at its connection with the vessel in which the hook-gauge Z was placed, and the arrangement was such that the height of the water in this vessel was at the mean level of the water 18 feet distant from the weirs on their down-stream side.

During the experiments the wheel was prevented from revolving, the brake being screwed up tight and securely blocked. The regulating-gate was opened to admit a quantity of water to give the desired depth on the weirs when there was no obstruction to the flow from the water on the down-stream side of the weirs; the quantity of water flowing over the weirs was then gauged by the usual weir formula; the level of the water on the down-stream side was then raised, a little at a time, and maintained long enough, at any desired height, to allow observations to be made at the several gauges. There is a waste weir in the canal, into which the water discharged over the weirs flowed, which afforded ample means for regulating the height of the water on the down-stream side of the weirs.

When the water is raised on the down-stream side of the weirs to a height to obstruct the flow over the weirs its effect is to raise the height

of the water on the up-stream side of the weirs without any change in the quantity of water flowing over the weirs, and the experiments consisted in observing the heights of the water above and below under these circumstances.

The data obtained by observation and the results of the computations are given in Table III, headed "Experiments on the Flow of Water over a Submerged Weir." Several of the columns require no further explanation than is contained in the headings. For brevity, I call the water on the down-stream side of the weir *backwater*.

Column 2, opening of the regulating-gate. This determined the quantity of water flowing over the weirs, and remained unchanged during a series of experiments with the backwater at different heights.

Column 6, fall at wheel. This is the difference in the heights of the water at the wheel-pit gauge in the box d and at the gauge f, and is the effective head acting upon the orifices formed by the opening of the regulating-gate given in column 2. If the head and orifices are constant the quantity of water discharged will also be constant. As has been stated, the orifices were constant during all the experiments in which the opening of the regulating-gate was the same; there were, however, variations in the head due to variations in the height in the forebay R, and also in the wheel-pit, the latter variations being due mainly to the obstruction to the flow over the weir caused by the backwater. The variations in the height in the forebay were due to variations in the height of the water in the canal from which the water was drawn. A correction in the depth on the weir is required for the variations in the head, as will be explained further on.

Column 7, mean elevation of water on down-stream side of the weirs, or the backwater, with reference to the crests of the weirs, by backwater hook gauge. These heights were observed at the hook-gauge in the vessel Z, and, as has been stated, is the height at the distance of 18 feet from the weirs. It was taken at this distance in order to be free from the disturbing effect due to the great agitation in the water caused by the flow over the weirs. Where blanks occur in this column the water was more than 0.75 feet below the crests of the weirs, and was too low to be observed by the hook-gauge.

Column 8, mean fall at wheel, and corresponding mean depth on weirs, when the water on the down-stream side of the weirs is at a height which it is assumed does not affect the flow over the weirs.

These are the standard falls and depths on the weirs, to which the observed depths on the weirs have been reduced on account of the variations in the head acting upon the orifices at the regulating-gate.

Column 9, mean depth on the weirs by observation. The depths were observed at the hook-gauge boxes XY in the manner described in Paper CCLXXXVIII for the experiments on the Humphrey turbine water-wheel.

Column 10, depth on weirs, on the assumption that the flow is not affected by the height of the water on the down-stream side, corrected for the fall at the wheel, by the formula

$$H' = H\left(\frac{h'}{h}\right)^{\frac{1}{3}}$$

in which H' = the corrected depth on the weirs.

H = the depth on the weirs when the fall acting on the wheel is h.

h' = the actual fall at the wheel.

To find the effect of the height of the backwater on the depth of the water on the weirs, it is necessary to know what the depth on the weirs would have been if there had been no backwater; this varies with the quantity flowing over the weirs, which varies with the head acting on the orifices in the regulating-gate. For comparison, I reduce the depths on the weirs to what they would have been if the effective head acting on the orifices in the regulating-gate had been constant for all the experiments with the same opening of gate, and as given in column 8.

Adopting the above notation, and calling m the co-efficient of discharge at the orifices in the regulating-gate, A their aggregate area, and n the co-efficient of the usual formula for flow over a weir, we have for the discharge at the gate, when the head is h,

$$Q = m\ A\ \sqrt{2\,gh}$$

and when the head is h',

$$Q' = m\ A\sqrt{2\,gh'}.$$

By division,

$$\frac{Q'}{Q} = \sqrt{\frac{h'}{h}}$$

For the discharge at the weirs we have

$$Q = \tfrac{2}{3}\ n\ \sqrt{2\,g}\ L\ H^{\frac{3}{2}}.$$

$$Q' = \tfrac{2}{3}\ n\ \sqrt{2\,g}\ L\ H'^{\frac{3}{2}}$$

$$\frac{Q'}{Q} = \left(\frac{H'}{H}\right)^{\frac{3}{2}} = \sqrt{\frac{h'}{h}}$$

Squaring both sides, and extracting the cube roots,

$$\frac{H'}{H} = \left(\frac{h'}{h}\right)^{\frac{1}{2}}$$

Then $H' = H\left(\frac{h'}{h}\right)^{\frac{1}{2}}$ as above.

The depths on the weirs given in column 10 have been reduced by this formula. It is immaterial to what particular head acting on the orifices in the wheel the reduction is made, except that it should be near the actual head. I have taken the average observed head, when there was no obstruction from backwater, for each opening of the regulating-gate, and the corresponding average depth on the weirs.

Column 11, change in the depth on the weirs caused by the backwater. This is the difference in the depths on the weirs given in columns 9 and 10. The differences are given only for the experiments in which it is supposed there could be any effect on the depth on the weir by the backwater.

Column 12, quantity of water discharged at the orifices in the regulating-gate and passing over the weirs, computed by the usual weir formula for the case when there is no obstruction to the flow from backwater. These quantities are assumed to be the actual quantities of water flowing over the weirs in the several experiments, whether the flow was affected by backwater or not.

Column 13, quantity of water discharged by the formula

$$Q' = 3.33\, L\,(H'' - H''')^{\frac{3}{2}} + 4.5988\, LH'''\sqrt{H'' - H'''}$$

which is adopted as the result of these experiments. A simpler form for computation, deduced directly from the preceding, is

$$Q' = 3.33\, L\sqrt{H'' - H'''}\,(H'' + 0.381\, H''')$$

The first term of the second member of the former equation is the formula for the flow over regular weirs without end contractions determined from the experiments given in Lowell Hydraulic Experiments. The co-efficient 4.5988 of the second term is determined from the 24 experiments selected from those given in the annexed table headed "Experiments on the Flow of Water over a Submerged Weir." The experiments selected are given in the following Table I. The above formula was arrived at after many trials, and represents very nearly the average result of the 24 experiments.

Putting the above formula in the form,

$$Q' = \tfrac{2}{3}\, m \sqrt{2g}\, L\,(H'' - H''') + n\, L\, H''' \sqrt{2g} \sqrt{H'' - H'''}$$

m, by Lowell Hydraulic Experiments, page 145, equals 0.6228; n, as deduced from the 24 selected experiments, equals 0.5734.

The values deduced from the several groups of experiments are as follows:

From experiments	Nos. 21 to 25,	$n = 0.5757$
"	" 43 to 47,	$n = 0.5747$
"	" 68 to 73,	$n = 0.5723$
"	" 80 to 82,	$n = 0.5792$
"	" 93 to 97,	$n = 0.5679$

The values of the co-efficients C and C', in the formula adopted, are found as follows, viz.: For the first term of the second member $C = \frac{2}{3}\, m \sqrt{2g} = \frac{2}{3} \times 0.6228 \times \sqrt{2g} = 3.33$; and for the second term of the second member $C' = n \sqrt{2g} = 0.5734 \sqrt{2g} = 4.5988$.

In several of the experiments in which the backwater was near the level of the crests of the weirs, it will be seen by the table that, instead of obstructing the flow, the backwater facilitated it; this is due to the change in the form of the sheet depending on whether there was air under it or not. In the column of remarks in Table III the condition of the sheet of water in this respect is given. When there is no air under the sheet, an action, called by some writers "the lateral communication of motion in fluids," comes into play; the increased flow in Venturi's tubes and in cylindrical ajutages appears to be due to this action. I had previously observed this effect in some experiments on a weir made in 1848, and reported in Lowell Hydraulic Experiments.

In Paper No. CCLIII of the Transactions of the Society (1883), by Messrs. A. Fteley and F. P. Stearns, presented at the Annual Convention, May 19, 1882, some experiments upon the discharge of water over a submerged weir are described, and a formula, based mainly upon them, is given. The volume of water discharged was much less than in the experiments reported in this paper, and the apparatus and method of conducting the experiments were very different. As both sets of experiments and the formulas proposed are for the same object, it is thought that a comparison of the results of the two formulas would be interesting.

Table I gives the computed quantities by both formulas, and also the quantity which is assumed to be the true quantity, in the 24 experi-

ments selected as above stated. It will be seen that the Fteley and Stearns formula gives results conforming to what are assumed to be the true quantities, very nearly as closely as the formula deduced directly from the same 24 experiments. This result was not to be expected, as the Fteley and Stearns formula is deduced from experiments in which the greatest depth on the weirs is less than the smallest depth in the experiments to which the formula is applied in Table I.

Table II is a similar comparison of the two formulas as applied to the experiments of Fteley and Stearns, given in Table XXV of their paper. As might be expected, their formula, with its variable co-efficient C, which is taken from their table XXVI, for each experiment, agrees closer with the results of their experiments than the proposed formula deduced from the experiments recorded in this paper.

Recapitulation.

In the formulas given above, the notation for the depth on the weirs and for the height of the backwater above the crest of the weirs, conforms to the notation in paper No. CCLXXXVIII of the Transactions of this Society, for convenience of reference. For greater simplicity in the formulas finally adopted as the results of these experiments, I call

H = the height of the water on the up-stream side of the weir above its crest, taken far enough from the weir to be unaffected by the curvature in the surface caused by the discharge.

H' = the height of the backwater, or the height of the water on the down-stream side of the weir above its crest, taken at a sufficient distance from the weir to be unaffected by the agitation caused by the flow over the weir.

Retaining the same notation as before for the co-efficients and the other quantities, the general formula for the discharge over a weir in which there is no contraction at the ends, is

$$Q = \tfrac{2}{3}\, m \sqrt{2g}\, L\, (H - H')^{\frac{3}{2}} + m'\, LH' \sqrt{2g\,(H - H')} \qquad (1)$$

As deduced from these experiments:

$m = 0.6228$,

$m' = 0.5734$,

$2g = 64.3236$, being the value at the place where the experiments were made.

Substituting these values in (1), we have

TABLE I.

1	2	3	4	5	6	7
Number of the Experiment.	Mean Depth on the Weir by Observation. Feet. H''	Mean Height of the Backwater. Feet. H'''	Quantity of Water flowing over the Weir. Cub. ft. per sec. Q	Value of C, by Fteley and Stearns' Table. C	Quantity of Water flowing over the Weir, computed by the Fteley and Stearns formula. $Q' = CL\left(H'' + \frac{H'''}{2}\right)\sqrt{H''-H'''}$ Cub. ft. per sec. Q'	Quantity of Water flowing over the Weir, computed by the proposed formula. $Q'' = 3.33\,L\,(H''-H''')^{\frac{3}{2}} + 4.5988\,L\,H'''\sqrt{H''-H'''}$ Cub. ft. per sec. Q''
21	1.203	+ 0.207	95.12	3.308	95.79	94.57
22	1.227	+ 0.309	95.00	3.247	95.47	95.25
23	1.277	+ 0.478	94.88	3.168	95.28	96.42
24	1.391	+ 0.860	94.64	3.090	91.05	92.58
25	1.491	+ 1.039	94.16	3.092	92.81	93.77
43	1.720	+ 0.466	159.25	3.233	157.04	157.09
44	1.740	+ 0.465	159.25	3.236	160.05	160.03
45	1.743	+ 0.483	158.97	3.229	159.64	159.91
46	1.804	+ 0.792	159.83	3.135	154.07	156.60
47	1.917	+ 0.996	159.25	3.107	159.94	162.93
68	1.994	+ 0.327	203.16	3.315	205.05	202.21
69	2.034	+ 0.528	203.01	3.241	202.86	202.78
70	2.092	+ 0.730	203.32	3.183	202.62	204.49
71	2.090	+ 0.732	203.47	3.182	202.16	204.08
72	2.188	+ 1.054	202.54	3.118	200.18	203.86
73	2.190	+ 1.071	202.54	3.116	199.44	203.17
80	2.212	+ 0.727	225.29	3.195	222.55	224.23
81	2.319	+ 1.111	224.17	3.119	218.74	222.81
82	2.318	+ 1.102	224.17	3.121	219.25	223.19
93	1.037	+ 0.263	73.48	3.246	74.09	73.96
94	1.091	+ 0.448	73.37	3.149	73.67	74.80
95	1.156	+ 0.657	73.15	3.096	72.14	73.44
96	1.149	+ 0.636	73.04	3.099	72.35	73.67
97	1.328	+ 1.015	72.71	3.107	70.83	70.92

TABLE II.

1	2	3	4	5	6
Number of the Experiment.	Mean Depth on the Weir by Observation. Feet. H''	Mean Height of the Backwater. Feet. H'''	Quantity of Water flowing over the Weir. $L = 1$ foot. Cub. ft. per sec. Q	Quantity of Water flowing over the Weir, computed by Fteley and Stearns' formula. $Q' = C\,L\left(H'' + \frac{H'''}{2}\right)\sqrt{H'' - H'''}$ $L = 1$ foot. Cub. ft. per sec. Q'	Quantity of Water flowing over the Weir, computed by the proposed formula. $Q'' = 3.33\,L\,(H'' - H''')^{\frac{3}{2}} + 4.5988\,L\,H'''\,\sqrt{H'' - H'''}$ $L = 1$ foot. Cub. ft. per sec. Q''
1	0.397	− 0.025	———	———	———
2	0.578	+ 0.010	1.465	1.462	1.460
3	0.396	+ 0.016	0.834	0.833	0.825
4	0.662	+ 0.033	1.810	1.810	1.781
5	0.574	+ 0.046	1.468	1.464	1.432
6	0.508	+ 0.073	1.203	1.196	1.176
7	0.592	+ 0.112	1.466	1.480	1.464
8	0.416	+ 0.118	0.834	0.835	0.838
9	0.325	+ 0.096	0.562	0.575	0.576
10	0.610	+ 0.186	1.467	1.469	1.476
11	0.716	+ 0.266	1.808	1.806	1.826
12	0.546	+ 0.209	1.202	1.195	1.209
13	0.633	+ 0.261	1.464	1.465	1.487
14	0.450	+ 0.219	0.834	0.839	0.854
15	0.486	+ 0.308	0.834	0.833	0.848
16	0.812	+ 0.516	1.806	1.801	1.827
17	0.742	+ 0.519	1.463	1.466	1.478
18	0.545	+ 0.419	0.834	0.835	0.833
19	0.428	+ 0.337	0.562	0.561	0.559
20	0.719	+ 0.573	1.199	1.200	1.193
21	0.625	+ 0.589	0.562	0.559	0.537
22	0.815	+ 0.795	0.562	0.567	0.526

$$Q = 3.33\ L\ (H - H')^{\frac{3}{2}} + 4.5988\ LH'\ \sqrt{H - H'} \qquad (2)$$

which, for convenience of computation, may be reduced to

$$Q = 3.33\ L\ \sqrt{H - H'}\ (H + 0.381\ H') \qquad (3)$$

Formulas (1), (2) and (3) are for the case where the weir is of the same length as the width of the channel leading to it; that is, where there is no contraction at the ends; if there is contraction at the ends, $L - 0.1\ n\ H$, in which n = the number of end contractions, must be substituted for L, and formula (3) becomes

$$Q = 3.33\ (L - 0.1\ n\ H)\ \sqrt{H - H'}\ (H + 0.381\ H') \qquad (4)$$

In the application of these formulas, the precautions described in article 164 of Lowell Hydraulic Experiments should be observed.

1	2	3	4				5	6	7	
			Time.							
No. of the experiment.	Opening of the regulating gate.	Date.	Beginning of the experiment.		Ending of the experiment.		Duration of the experiment.	Fall at wheel. h'	Mean elevation of water on down-stream side of the weirs, with reference to the crests of the weirs, by Backwater Hook Gauge. H''	Mean depth stream it is as weirs.
	Inches.	1883.	Hour.	Min.	Hour	Min.	Minutes.	Feet.	Feet.	Fall
1	3.56	Oct. 31, A. M.	9	17	9	20	3	13.863	—0.448	
2	"	" " "	9	46	9	51	5	13.876	—0.638	
3	"	" " "	10	10	10	13	3	13.903	—0.476	
4	"	" " "	10	20	10	26	6	13.906	—0.374	
5	"	" " "	10	34	10	39	5	13.907	—0.310	
6	"	" " "	10	50	10	55	5	13.905	—0.230	
7	"	" " "	11	3	11	5	2	13.896	—0.121	
8	"	" " "	11	14	11	17	3	13.913	—0.028	
9	"	" " "	11	22	11	25	3	13.933	+0.032	
10	"	" " "	11	31	11	36	5	13.928	+0.134	
11	"	" " "	11	41	11	45	4	13.921	+0.249	
12	"	" " "	11	47	11	50	3	13.928	+0.227	
13	"	" " P. M.	2	26	2	30	4	13.929	—0.318	
14	"	" " "	2	35	2	39	4	13.911	—0.251	
15	"	" " "	2	40	2	43	3	13.901	—0.238	
16	"	" " "	2	46	2	49	3	13.903	—0.155	
17	"	" " "	2	59	3	4	5	13.938	—0.009	
18	"	" " "	3	5	3	7	2	13.949	—0.022	
19	"	" " "	3	12	3	16	4	13.945	+0.096	
20	"	" " "	3	17	3	20	3	13.943	+0.089	
21	"	" " "	3	26	3	32	6	13.930	+0.207	
22	"	" " "	3	41	3	43	2	13.903	+0.309	
23	"	" " "	3	50	3	53	3	13.860	+0.478	
24	"	" " "	4	10	4	13	3	13.785	+0.860	
25	"	" " "	4	33	4	35	2	13.663	+1.039	
26	"	" " "	4	54	4	58	4	13.925	—0.638	
27	"	" " "	4	59	5	4	5	13 918	.....	
28	"	" " "	5	5	5	11	6	13.917	.....	
29	"	" " "	5	14	5	18	4	13.919	.....	
30	"	" " "	5	19	5	24	5	13.913	.....	
31	"	" " "	5	25	5	29	4	13.920	.....	
32	6.74	Nov. 3, A. M.	8	36	8	41	5	13.223	.....	
33	"	" " "	8	42	8	47	5	13.225	.....	
34	"	" " "	8	52	8	56	4	13.219	.....	
35	"	" " "	8	57	9	1	4	13.195	.....	
36	"	" " "	9	9	9	13	4	13.171	—0.553	
37	"	" " "	9	27	9	29	2	13 160	—0.323	
38	"	" " "	9	37	9	39	2	13.143	—0.249	
39	"	" " "	9	45	9	52	7	13.136	—0.083	
40	"	" " "	9	55	9	58	3	13.128	+0.025	
41	"	" " "	9	59	10	2	3	13.133	+0.022	
42	"	" " "	10	10	10	14	4	13.123	+0.075	
43	"	" " "	10	28	10	30	2	13.063	+0.466	
44	"	" " "	10	31	10	33	2	13.043	+0.465	
45	"	" " "	10	38	10	41	3	13.016	+0.483	
46	"	" " "	10	55	10	57	2	13.147	+0.792	
47	"	" " "	11	9	11	14	5	13.060	+0.996	
48	"	" " "	11	22	11	26	4	13.265	—0.143	
49	"	" " "	11	30	11	33	3	13.258	.....	
50	"	" " "	11	34	11	40	6	13.247	.....	
51	"	" " "	11	43	11	45	2	13.223	—0.738	

TABLE III.

RIMENTS ON THE FLOW OF WATER OVER A SUBMER

7	8		9	10
ean elevation of water on own-stream e of the weirs, ith reference the cresta of e weirs, by kwater Hook Gauge. H''	Mean fall at wheel and corresponding mean depth on weirs when the water on the down-stream side of the weirs is at a height which it is assumed does not affect the flow over the weirs.		Mean depth on the weirs by observation.	Depth on weirs on the assumption that the flow is not affected by the height of the water on the down-stream side, corrected for the fall at the wheel by the formula, $H'=H\left(\frac{h'}{h}\right)^{\frac{1}{2}}$
	Fall at wheel. h	Depth on weirs. H	H''	H'
Feet.	Feet.	Feet.	Feet.	Feet.
—0.448	13.906	1.1824	1.175	1.181
—0.638	"	"	1.175	1.181
—0.476	"	"	1.182	1.182
—0.374	"	"	1.177	1.182
—0.310	"	"	1.177	1.182
—0.230	"	"	1.177	1.182
—0.121	"	"	1.182	1.182
—0.028	"	"	1.175	1.183
+0.032	"	"	1.175	1.183
+0.134	"	"	1.179	1.183
+0.249	"	"	1.203	1.183
+0.227	"	"	1.199	1.183
—0.318	"	"	1.185	1.183
—0.251	"	"	1.185	1.182
—0.238	"	"	1.185	1.182
—0.155	"	"	1.183	1.182
—0.009	"	"	1.186	1 183
—0.022	"	"	1.186	1.184
+0.096	"	"	1.183	1.183
+0.089	"	"	1.182	1.183
+0.207	"	"	1.203	1.183
+0.309	"	"	1.227	1.182
+0.478	"	"	1.277	1.181
+0.860	"	"	1.391	1.179
+1.039	"	"	1.491	1.175
—0.638	"	"	1.186	1.183
.....	"	"	1.186	1.183
.....	"	"	1.186	1.183
.....	"	"	1.187	1.183
.....	"	"	1.186	1.183
.....	"	"	1.185	1.183
.....	13.225	1.6754	1.675	1.675
.....	"	"	1.675	1.675
.....	"	"	1.676	1.675
.....	"	"	1.674	1.674
—0.553	"	"	1.674	1.673
—0.323	"	"	1.672	1.673
—0.249	"	"	1.672	1.672
—0.083	"	"	1.673	1.672
+0.025	"	"	1.670	1.671
+0.022	"	"	1.670	1.671
+0.075	"	"	1.670	1.671
+0.466	"	"	1.720	1.668
+0.465	"	"	1.740	1.668
+0.483	"	"	1.743	1.666
+0.792	"	"	1.804	1.672
+0.996	"	"	1.917	1.668
—0.143	"	"	1.679	1.677
.....	"	"	1.675	1.677
.....	"	"	1.676	1.676
—0.738	"	"	1.675	1.675

	11	12	13	14
n the as-ow is not ht of the n-stream he fall at mula, $\frac{1}{3}$	Change in the depth on the weirs caused by the height of the water on the down-stream side of the weirs.	Quantity of water discharged, by the formula, $Q = 3.33\ L\ H^{\frac{3}{2}}$	Quantity of water discharged, by the formula, $Q' =$ $3.33\ L\left(H'' - H'''\right)^{\frac{3}{2}} +$ $4.5988\ L\ H'''\sqrt{H'' - H'''}$	[Remarks relating to air under the sheet of water passing over the weirs. The heights are the heights of the back-water above or below the crests of the weirs.
	Feet.	Cubic feet per second.	Cubic feet per second.	
		94.88		
		94.88		
		95.00		
		95.00		
		95.00		
		95.00		
		95.00		
		95.12		
	−0 008	95.12		
	−0.004	95.12		
	+0.020	95.12	93.71	
	+0 016	95.12	93.69	
		95.12		
		95 00		+ 0.01 ft. air began to disappear.
		95.00		+ 0.09 ft. air had all disappeared.
		95.00		Between these heights of backwater,
		95.12		air under sheet irregular; flashing in
		95.24		and out for longer or shorter distances.
	0.000	95.12		
	−0.001	95.12		
	+0.020	95.12	94.57	
	+0.045	95.00	95.25	
	+0.096	94.88	96.42	
	+0.212	94.64	92.58	
	+0 316	94.16	93.77	
		95.12		
		95.12		
		95.12		
		95.12		
		95.12		
		95.12		
		160.26		
		160.26		
		160.26		
		160.11		
		159.97		
		159.97		
		159.83		
		159.83		
	−0.002	159.68	159.24	− 0.10 ft. air began to disappear.
	−0.002	159.68	159.28	+ 0 08 ft. air had all disappeared.
	−0.001	159.68	158.58	Between these heights, same remarks
	+0.052	159.25	157.09	as above.
	+0.073	159.25	160.03	
	+0.076	158.97	159.91	
	+0.132	159.83	156.60	
	+0.249	159.25	162.93	
		160.55		
		160.55		
		160.40		
		160.26		

1	2	3	4				5	6	7	
			Time.							
No. of the experiment.	Opening of the regulating gate.	Date.	Beginning of the experiment.		Ending of the experiment.		Duration of the experiment.	Fall at wheel. h'	Mean elevation of water on down-stream side of the weirs, with reference to the crests of the weirs, by Backwater Hook Gauge. H'''	Mean depth o stream it is ass weirs. Fall
	Inches.	1883.	Hour.	Min.	Hour.	Min.	Minutes.	Feet.	Feet.	
52	9.84	Nov. 5, P. M.	2	8	2	12	4	12.625	....	
53	"	" " "	2	13	2	18	5	12.626	....	
54	"	" " "	2	22	2	26	4	12.623	—0.743	
55	"	" " "	2	27	2	30	3	12.624	—0.726	
56	"	" " "	2	36	2	39	3	12.613	—0.557	
57	"	" " "	2	40	2	43	3	12.615	—0.553	
58	"	" " "	2	48	2	52	4	12.609	—0.385	
59	"	" " "	2	53	2	57	4	12.607	—0.352	
60	"	" " "	3	2	3	6	4	12.597	—0.127	
61	"	" " "	3	7	3	10	3	12.596	—0.133	
62	"	" " "	3	13	3	16	3	12.591	—0.069	
63	"	" " "	3	17	3	20	3	12.588	—0.049	
64	"	" " "	3	24	3	27	3	12.570	+0.029	
65	"	" " "	3	28	3	31	3	12.573	+0.049	
66	"	" " "	3	33	3	36	3	12.558	+0.128	
67	"	" " "	3	37	3	41	4	12.549	+0.173	
68	"	" " "	3	47	3	52	5	12.523	+0.327	
69	"	" " "	4	1	4	4	3	12.493	+0.528	
70	"	" " "	4	12	4	14	2	12.537	+0.730	
71	"	" " "	4	15	4	18	3	12.553	+0.732	
72	"	" " "	4	31	4	34	3	12.450	+1.054	
73	"	" " "	4	35	4	39	4	12.449	+1.071	
74	"	" " "	4	52	4	58	6	12.618	—0.717	
75	"	" " "	4	59	5	5	6	12.601	—0.672	
76	"	" " "	5	8	5	12	4	12.609	—0.635	
77	12.74	Nov. 6, A. M.	8	31	8	35	4	12.273	.....	
78	"	" " "	8	42	8	45	3	12.260	.....	
79	"	" " "	8	55	8	59	4	12.253	.....	
80	"	" " "	10	25	10	30	5	12.086	+0.727	
81	"	" " "	10	40	10	43	3	11.953	+1.111	
82	"	" " "	10	44	10	48	4	11.955	+1.102	
83	"	" " "	11	2	11	5	3	12.413	—0.483	
84	"	" " "	11	10	11	14	4	12.401	—0.521	
85	2.73	Nov. 6, P. M.	2	30	2	34	4	14.063	.....	
86	"	" " "	2	35	2	43	8	14.050	.....	
87	"	" " "	2	46	2	51	5	14.025	—0.687	
88	"	" " "	2	58	3	4	6	14.006	—0.543	
89	"	" " "	3	20	3	24	4	13.959	—0.144	
90	"	" " "	3	30	3	33	3	13.956	—0.066	
91	"	" " "	3	41	3	44	3	13.918	+0.052	
92	"	" " "	3	46	3	51	5	13.921	+0.136	
93	"	" " "	3	58	4	1	3	13.858	+0.263	
94	"	" " "	4	8	4	14	6	13.800	+0.448	
95	"	" " "	4	21	4	23	2	13.719	+0.657	
96	"	" " "	4	25	4	27	2	13.706	+0.636	
97	"	" " "	4	41	4	46	5	13.545	+1.015	
98	"	" " "	4	59	5	4	5	13.796	.....	
99	"	" " "	5	5	5	10	5	13.786	—0.724	
100	"	" " "	5	12	5	16	4	13.767	—0.739	
101	"	" " "	5	17	5	21	4	13.755	—0.715	

TABLE III—(*Continued*).

IMENTS ON THE FLOW OF WATER OVER A SUBMERGED WEIR.

7	8		9	10	11	12	13	14
n elevation water on n-stream f the weirs, reference e crests of weirs, by ater Hook Gauge. H'''	Mean fall at wheel and corresponding mean depth on weirs when the water on the down-stream side of the weirs is at a height which it is assumed does not affect the flow over the weirs.		Mean depth on the weirs by observation.	Depth on weirs on the assumption that the flow is not affected by the height of the water on the down-stream side, corrected for the fall at the wheel by the formula, $H=H'\left(\frac{h'}{h}\right)^{\frac{1}{2}}$	Change in the depth on the weirs caused by the height of the water on the down-stream side of the weirs.	Quantity of water discharged, by the formula, $Q=3.33\,L\,H^{\frac{3}{2}}$	Quantity of water discharged, by the formula, $Q=3.33\,L\left(H''-H'''\right)^{\frac{3}{2}}+4.5988\,L\,H'''\sqrt{H''-H'''}$	Remarks relating to air under the sheet of water passing over the weirs. The heights are the heights of the back-water above or below the crests of the weirs.
	Fall at wheel. h	Depth on weirs. H	H''	H'				
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Cubic feet per second.	Cubic feet per second.	
.....	12.617	1.9671	1.968	1.967	.....	203.94	.....	
.....	"	"	1.967	1.968	.....	204.10	.....	
—0.743	"	"	1.968	1.967	.....	203.94	.....	
—0.726	"	"	1.966	1.967	.....	203.94	.....	
—0.557	"	"	1.967	1.967	.....	203.94	.....	
—0.553	"	"	1.966	1.967	.....	203.94	.....	
—0.385	"	"	1.968	1.967	.....	204.94	.....	
—0.352	"	"	1.968	1.967	.....	203.94	.....	
—0.127	"	"	1.965	1.966	.....	203.79	.....	
—0.133	"	"	1.968	1.966	.....	203.79	.....	
—0.069	"	"	1.963	1.966	.....	203.79	.....	
—0.049	"	"	1.962	1.966	.....	203.79	.....	— 0.14 ft. air began to disappear.
+0.029	"	"	1.965	1.965	0.000	203.63	.....	+ 0.17 ft. air had all disappeared.
+0.049	"	"	1.965	1.965	+0.001	203.63	.....	Between these heights, same remarks as above.
+0.128	"	"	1.976	1.964	+0.012	203.47	.....203.48	
+0.173	"	"	1.976	1.964	+0.013	203.47	.....202.69	
+0.327	"	"	1.994	1.962	+0.032	203.16	.....202.21	
+0.528	"	"	2.034	1.961	+0.073	203.01	.....202.78	
+0.730	"	"	2.092	1.963	+0.129	203.32	.....204.49	
+0.732	"	"	2.090	1.964	+0.126	203.47	.....204.08	
+1.054	"	"	2.188	1.958	+0.230	202.54	.....203.86	
+1.071	"	"	2.190	1.958	+0.232	202.54	.....203.17	
—0.717	"	"	1.968	1.967	.....	203.94	.....	
—0.672	"	"	1.968	1.966	.....	203.79	.....	
—0.635	"	"	1.966	1.967	.....	203.94	.....	
.....	12.320	2.1159	2.114	2.113	.....	227.06	.....	
.....	"	"	2.113	2.112	.....	226.90	.....	
.....	"	"	2.113	2.112	.....	226.90	.....	— 0.30 ft. air began to disappear.
+0.727	"	"	2.212	2.102	+0.109	225.29	.....224.23	+ 0.10 ft. air had all disappeared.
+1.111	"	"	2.319	2.095	+0.224	224.17	.....222.81	Between these heights, same remarks as above.
+1.102	"	"	2.318	2.095	+0.223	224.17	.....223.19	
—0.483	"	"	2.119	2.121	.....	228.35	.....	
—0.521	"	"	2.120	2.120	.....	228.19	.....	
.....	13.906	0.9973	1.002	1.001	.....	74.04	.....	
.....	"	"	1.001	1.001	.....	74.04	.....	
—0.687	"	"	1.001	1.000	.....	73.93	.....	
—0.543	"	"	1.000	1.000	.....	73.93	.....	
—0.144	"	"	1.000	0.999	.....	73.82	.....	
—0.066	"	"	1.000	0.998	.....	73.70	.....	
+0.052	"	"	0.999	0.998	+0.001	73.70	.....73.30	0.00 ft. air began to disappear.
+0.136	"	"	1.002	0.998	+0.005	73.70	.....72.50	+ 0.10 ft. air had all disappeared.
+0.263	"	"	1.037	0.996	+0.041	73.48	.....73.96	Between these heights, same remarks as above.
+0.448	"	"	1.091	0.995	+0.096	73.37	.....74.80	
+0.657	"	"	1.156	0.993	+0.164	73.15	.....73.44	
+0.636	"	"	1.149	0.992	+0.157	73.04	..... 73.67	
+1.015	"	"	1.328	0.989	+0.339	72.71	.....70.92	
.....	"	"	0.994	0.995	.....	73.37	.....	
—0.724	"	"	0.994	0.994	.....	73.26	.....	
—0.739	"	"	0.994	0.994	.....	73.26	.....	
—0.715	"	"	0.993	0.994	.....	73.26	.....	